Towards a unified understanding of flow and jet energy loss at strong coupling.

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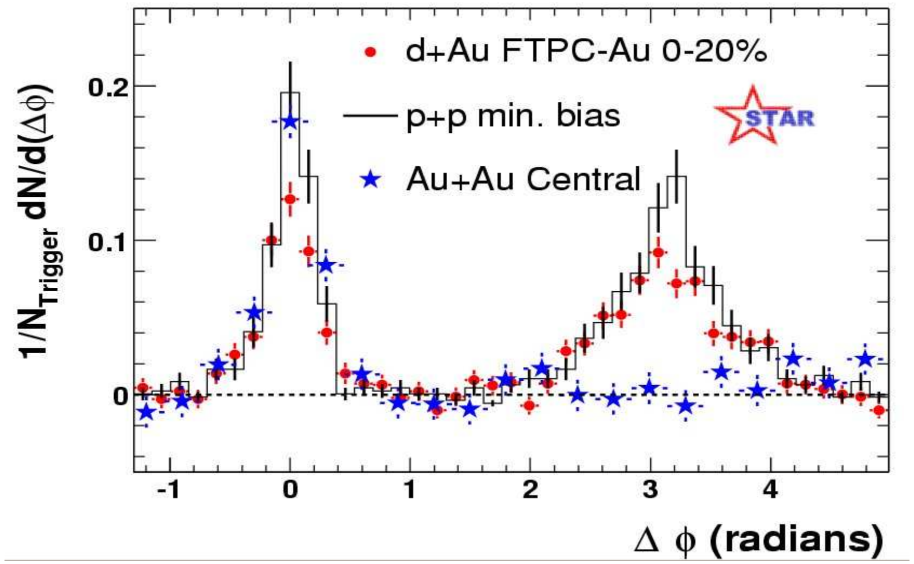
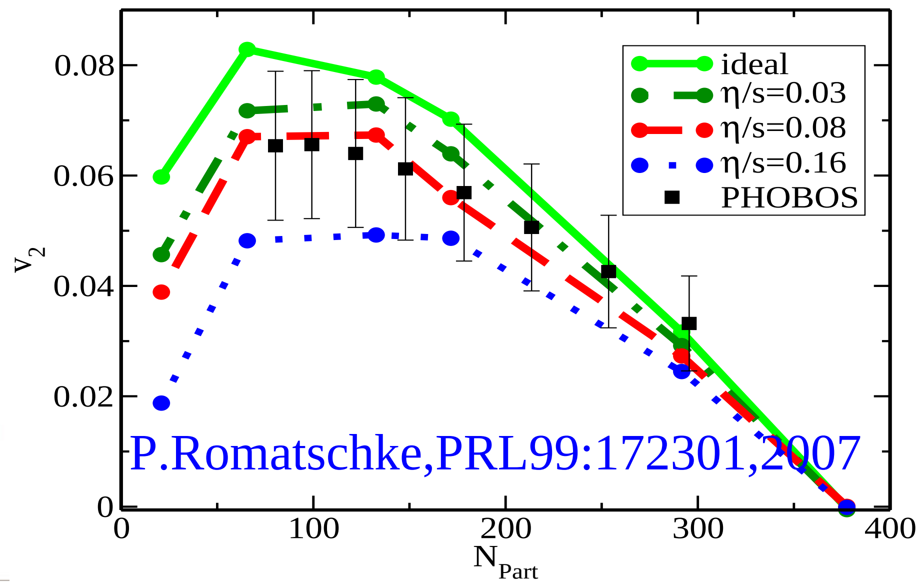
based on 0906.4099

RHIC: a short summary!





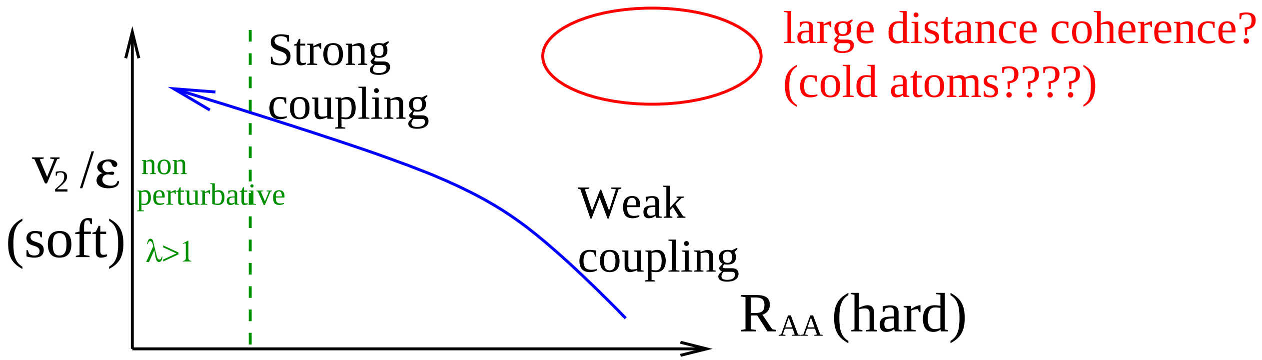
Matter created at RHIC is opaque! Matter created at RHIC is fluid!

Taken together, these MIGHT suggest a strongly coupled system

These two findings are likely, but not certain, to be correlated

viscosity Measures the strenght of the interaction of "medium" particles ($\langle q \rangle \sim T$) with each othe, close to equilibrium
Prevents gradients in initial density (eg the 2nd Fourier component ϵ_p) from transforming, via hydrodynamic pressure gradients, into gradiens of flow ($\epsilon_{\langle P \rangle} \sim v_2$).

Opacity Measures the strength of the interaction of "hard" particles ($\langle q \rangle >> T$, dominated by pQCD) with medium .
Brings momentum of hard partons down towards "medium", sometimes parametrized by $\hat{q} = \frac{s_{average \quad kick}}{l_{mfp}}$

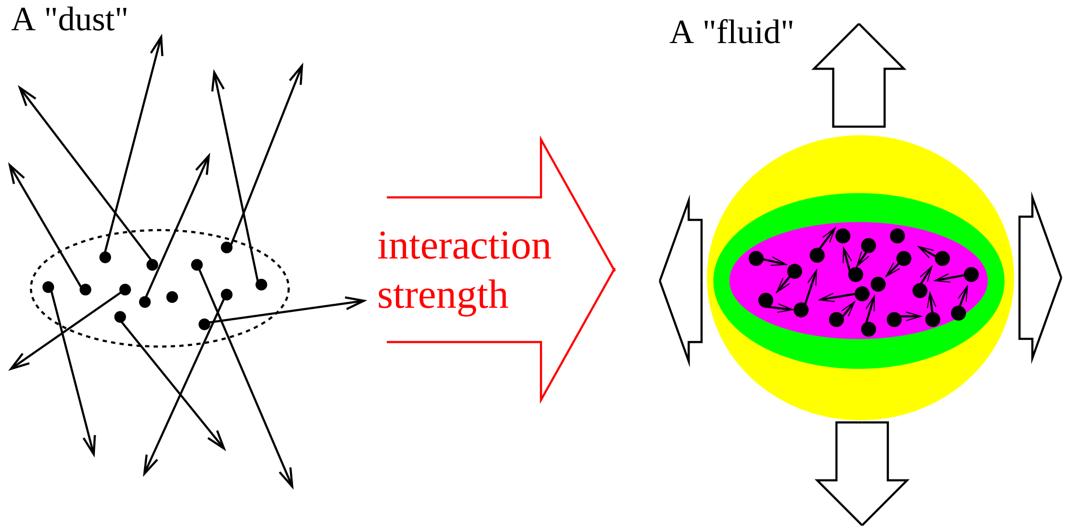


Neglecting the running of the coupling constant (one would need either non-perturbative physics or *very* high momentum exchange), therefore, η and $\hat{q}$ should be characterized by a similar coupling constant

As this constant goes up, viscosity decreases (v_2/ϵ goes up) and opacity increases (R_{AA},the scaled number of hard particles, "jets", decreases)

This "conceivably" continues in the non-perturbative regime, but we can not calculate it within QCD. Hence, for perturbative QCD to be valid, the system can not be too fluid/opaque; $\eta/s \geq\sim 1$

Hydrodynamics: An effective theory for "medium" ($v \sim T$) particles, has nothing to say for particles (ie jets) where $v \gg T$. Need to use microscopic theory to correlate viscosity and opacity. If $\lambda \ll 1$ the most ready way to answer this question is via the Boltzmann equation



Microscopic picture: Boltzmann equation
(local molecular chaos, $\langle f(x+l_{mfp}) f(x) \rangle \simeq \langle f(x+l_{mfp}) \rangle \langle f(x) \rangle$) :

$$\left(\frac{1}{m} p^\mu \frac{\partial}{\partial x^\mu} + F^\mu \frac{\partial}{\partial p^\mu} \right) f(x,p) = C^{2body}[f] + C^{3body}[f] + ...$$

$$C^{2body} = \int d^3[X, X', P, P'] \sigma(P, P' \Leftrightarrow p, p') \left[f(X,P) f(X',P') - f(x,p) f(X',P') \right]$$

Ideal hydro: $C = 0$ (Gain=Loss) $f = \Upsilon e^{-p_\mu u^\mu / T}$ always,(T, u_μ change)

Non-ideal: Expand $C[f]$ around $f - f_{eq}$,$\equiv$ Knudsen n.$K = l_{mfp} \partial_\mu u_\nu$

Approximate equilibrium $\leftrightarrow$ small Knudsen Number $K = l_{mfp}\partial_\mu u_\nu$ Ideal hydro $O(K^0)$,Navier-Stokes $O(K^1)$,Israel-Stewart $O(K^2)$. Note K "really" a "tensor". (Grad expansion):

$$f = f_{eq}\left[\frac{u^\mu p_\mu}{T}\right]\left[1 + \underbrace{\epsilon}_{O(K^1)[\zeta]+higher} + \underbrace{\epsilon_\mu}_{O(K^1)[\eta]+higher} p^\mu + \underbrace{\epsilon_{\mu\nu}}_{O(K^2)+higher} p^\mu p^\nu + ...\right]$$

Plug into Boltzmann equation use H-theorem and obtain ϵ_μ in terms of $\eta\partial u$ etc.. For first order, we can show that

$$\eta = \frac{1}{5}\langle p\rangle s l_{mfp} \quad , \quad \zeta = \left(c_s^2 - \frac{1}{3}\right)^2 \eta$$

Last relation relies on 1 reaction, broken if elastic and inelastic collisions equivalent to Kubo formulae in perturbative case!

Perturbatively, (constant λ)

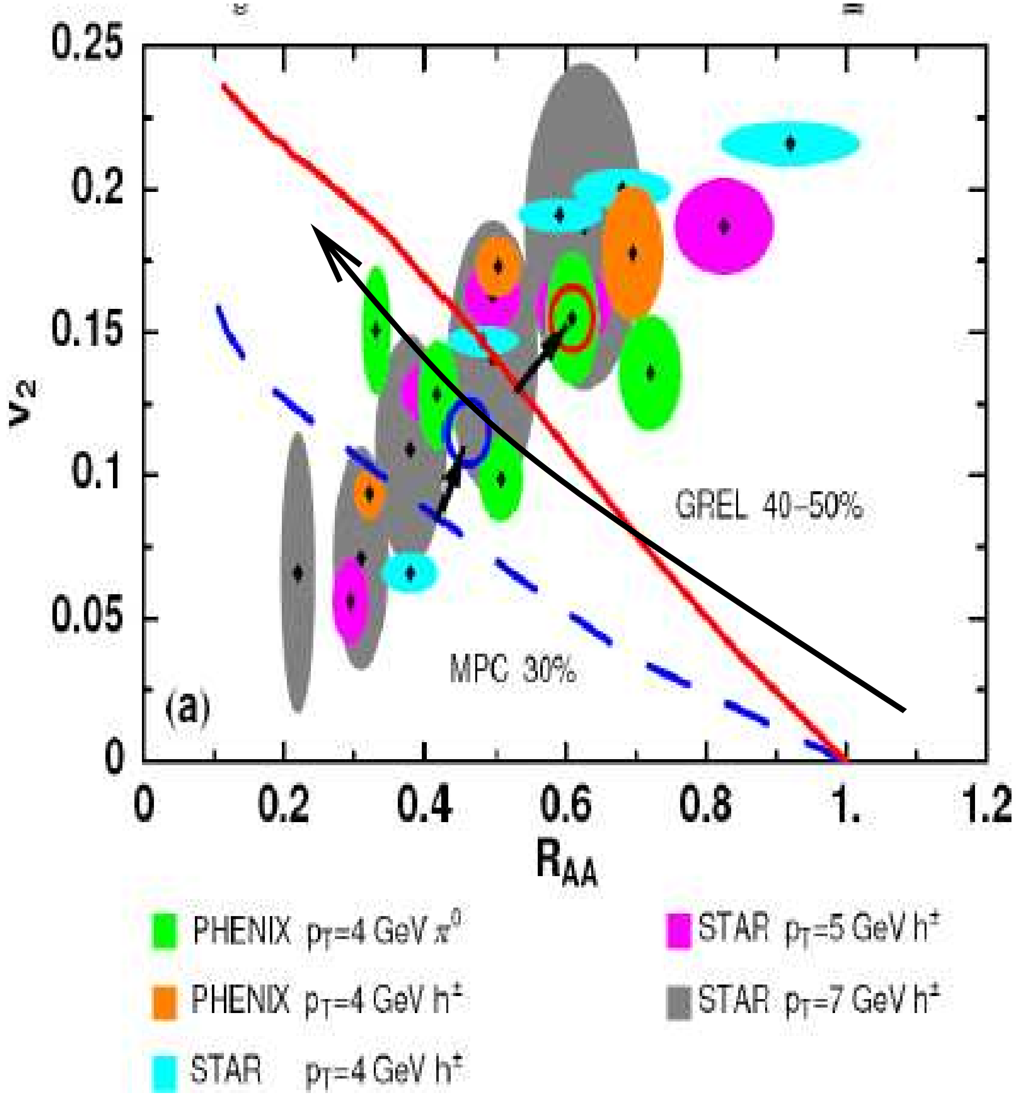
$$\frac{\hat{q}}{s} \sim \lambda \quad , \quad \frac{\eta}{s} \sim \frac{1}{\lambda^2 \ln \lambda}$$

Majumder,Mueller,Wang : Boltzmann estimate with light quark jets linking $\hat{q}, \eta/s$
Moore+Teaney, Phys.Rev.C71:064904,2005:Fokker-Plank solution for heavy quark drag, diffusion coefficient linked to viscosity

In both these cases, the opacity (weather parametrized by dE/dxor $\hat{q}$ depends inversely on viscosity, and directly on the coupling constant. A gas with low viscosity is also expected to be opaqueBarring large-scale quantum coherence (superfluidity etc) in the medium, this conclusion is reasonable and general.So, what is the coupling constant capable of explaining both η/s and opacity? For quantitative comparison need partonic transport study

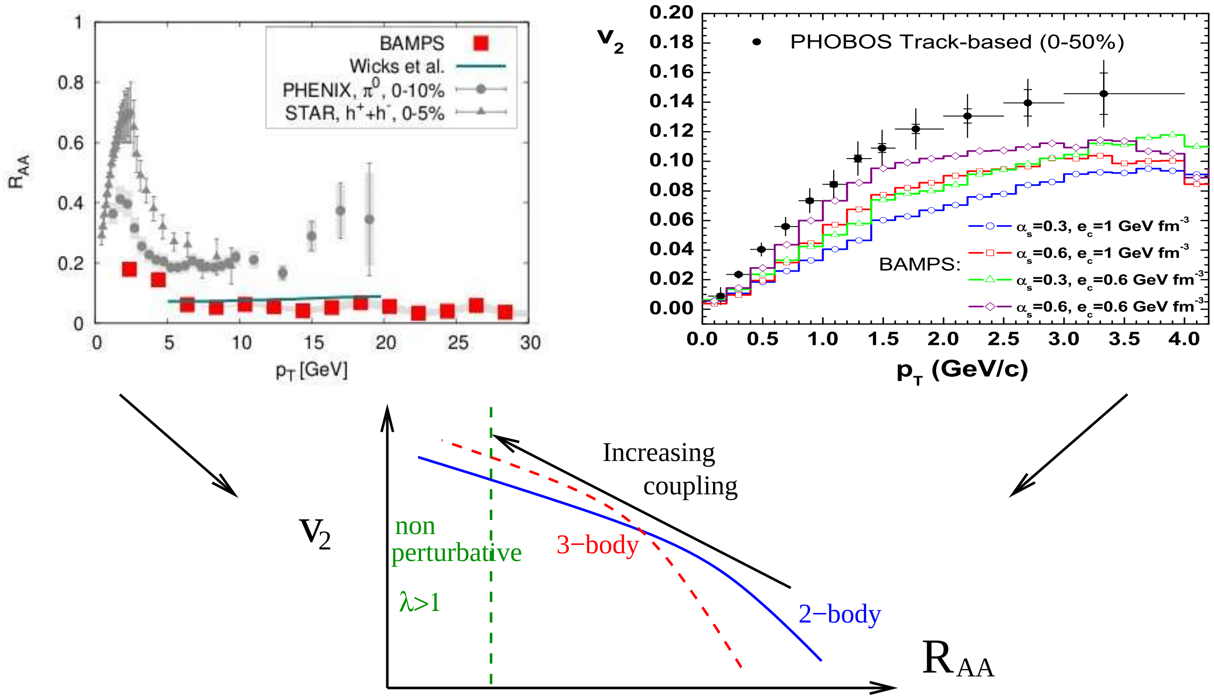
Gyulassy,Molnar,2001

Using kinetic model to describe both flow and jets seems impossible



Multi-parton interactions could save the day.

Z.Xu
C.Greiner
O.Fochler
I.Bouras
...



But remember again that Boltzmann equation is not fundamental, correlation terms thrown away. Strong coupling alternative still necessary for meaningful comparison

Beyond weak coupling I

What happens when coupling is strong (non-perturbative)?
In the non-perturbative limit

- We can not anymore use the Scattering approximation, and hence molecular chaos. Microscopic degrees of freedom are strongly correlated.

- 3 particle interactions will be more likely than 2-particle, 4 particle more likely than 3 particle and so on...

Hence the use of the Boltzmann equation not justified.

Is hydrodynamics justified at stong coupling?

The Euler,Naiver Stokes, and Israel-Stewart equations are simply the conservation of energy-momentum of a field, $\partial_\nu T^{\mu\nu} = 0$ together with a coarse-grained definition for $T_{\mu\nu}$ as the most general Lorentz-invariant dissipative expression for energy momentum that can be written in terms of $diag[p, \rho, \rho, \rho]$,u^μ and their gradients.

It can be considered an effective theory built around the idea that $K = \tau \partial_\mu \ll 1$, where τ is a "microscopic" equilibration time (More general than l_{mfp} , applies to any field where Entropy increases) and ∂ is the gradient (of p, ρ, u_μ). Euler simply keeps all terms order K^0 ,NS K^1 and "I-S" K^2

A breakdown of molecular chaos could change τ,but not introduce new terms in the NS equation, as theyd either violate Lorenz symmetry or the 2nd law.

Is hydrodynamics justified at strong coupling?

Some people regard hydrodynamics as a limiting theory of the Boltzmann equation (and hydrodynamics people as "too stupid/lazy to do transport"). not quite true:Hydrodynamics is a limit of the Boltzmann equation, but it also applies to many other systems. any system where

- The second law of thermodynamics and causality apply (system is local and entropy increases!)

- the equilibration time is small wrt evolution of the local density ($\sim K$ in weak coupling).

These requirements are more general than those satisfied by the Boltzmann equation. There are systems where hydrodynamics applies and the Boltzmann equation is lousy. eg Water!

How low can the viscosity be? Lets forget we cant use the Boltzmann equation at strong coupling!A rough estimate: (Danielewicz and Gyulassy)

$$l_{mfp} \geq \langle \lambda_{debroglie} \rangle \sim 1/\langle p \rangle$$

If one plugs this into the Boltzmann equations and calculates viscosity the usual way, a lower limit is obtained

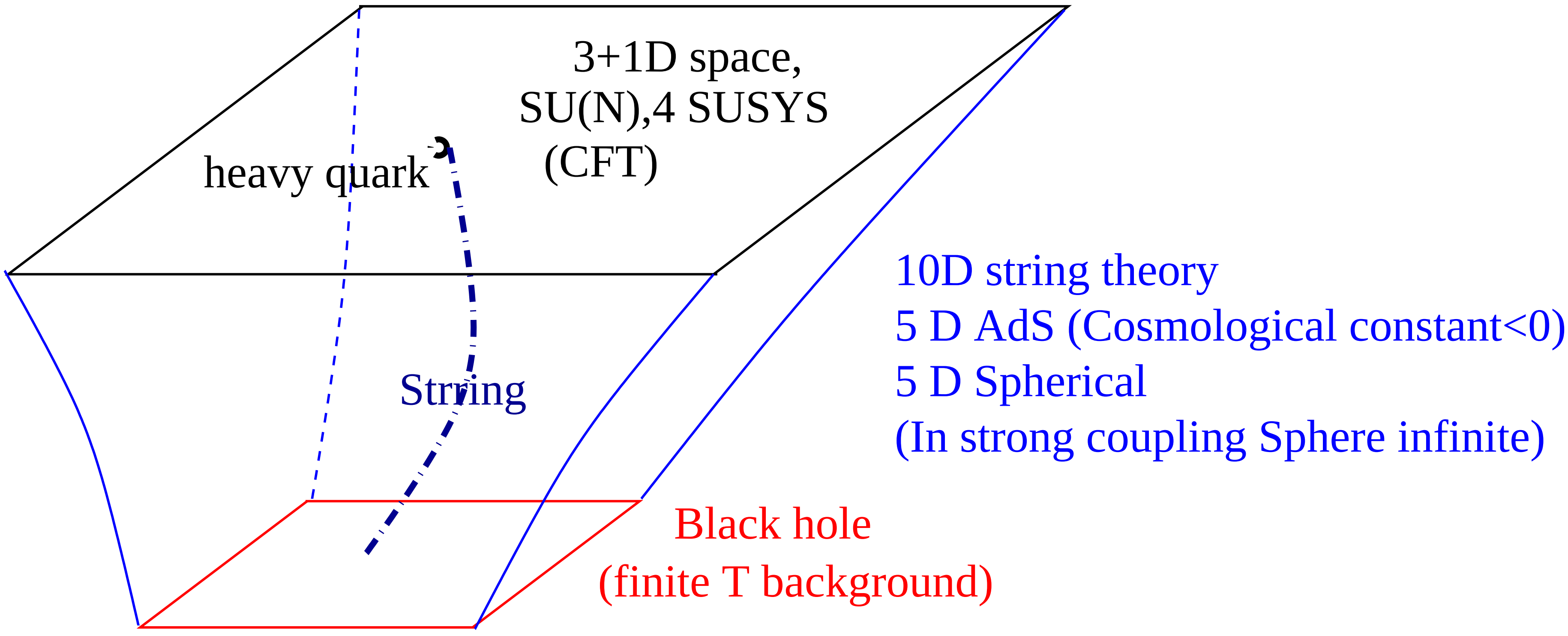
$$\eta/s \geq 1/(15\pi)$$

but this procedure is less than rigurous:Remember, we cant use Boltzmann!

A way to make this (a bit!) more rigurous: Hydrodynamics (and viscosity) from AdS/CFT

The AdS-CFT correspondence: Every $\left\langle \hat{O}_{CFT} \right\rangle$ a 4D $N_{susy} = 4$ Gauge theory with N_c colors and T'hooft coupling λ , can be calculated by translating to a 10D string theory, with 5 Anti-DeSitter ($\Lambda < 0$) dimensions, 5 dimensions compactified on a sphere, and a string coupling constant of $g_s = \lambda/(4\pi N_c)$

- dictionary between $\hat{O}_{CFT}$ and $\hat{O}_{ADS}$ can be worked out
- Links strongly coupled CFT to weakly coupled perturbative string theory. Infinitely strongly coupled CFT $\Leftrightarrow$ classical supergravity.



$g_{\mu\nu}|_{asymptotic} \Leftrightarrow T_{\mu\nu}$

Finite T background $\Leftrightarrow$ Black hole in AdS space

$\lambda \to \infty$ $\Leftrightarrow$ Classical geometry (Einstein's equations for $g^{\mu\nu}$)

A BIG note of caution: This is NOT QCD (4 SUSYs, no quarks, $N_c, \lambda \to \infty$). This has the potential of introducing qualitative subtle differences.

CFT The theory is conformally invariant. No running coupling, no phase transition, no hadrons, no bulk viscosity

QCD Is approximately conformally invariant at weak coupling, big-time non-invariant at strong coupling

But we just want to check that hydrodynamics works in a strongly coupled theory, so thats OK as a "toy-model" (still: CFT is a symmetry QCD does not have. And its a conjecture. So Caveat Emptor!).

This way we can describe both hydrodynamics and jets!

Linearized Hydro (EoS,viscosity,relaxation time...) corresponds to the dynamics of a "slightly perturbed" black hole in 5 dimensions, corresponding to the given Hawking temperature
Note that, unlike in flat space, AdS black hole have a thermal equilibrium radius wrt the vacuum

Drag force on Jet in a medium Corresponds to the general relativistic problem of a string attached to the black hole (the medium) being dragged along the 5th dimension. Only solvable analytically for infinitely heavy quarks (numerical solutions for lighter quarks in development), and a dragged string (no deceleration. Probably good if deceleration $\ll T$

Entropy density Can be extracted from the entropy of the Black hole: $s = \frac{3}{4} s_{SB}$

η Can be gotten with the Kubo formula, via the linearized theory of perturbations of a Black hole in AdS-space $\eta \sim \lim_{w \to 0} e^{iwx} \langle h_{\mu\nu}(0) h_{\mu\nu}(x) \rangle$. Plugging in the numbers we get the famous "limit"

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

(Compare with Kinetic theory limit of $1/15\pi$).

NB: It seems the bound is violated for more complicated dual theories. not clear if η/s can go to 0.

Hydrodynamics can be investigated by perturbations on the black hole. It seems that strongly coupled system can indeed be described by Israel-Stewart equations (Janik,Peshanski,Kovchegov,Minwalla,...). All coefficients compatible with CFT worked out (Baier,Romatschke,Son,...)! Usual hydrodynamic phenomena (Sound waves, Mach cones) are there and are very similar to expectations from Navier-Stokes equations (eg Chesler+Yaffee,Yarom+Pufu+Gubser,Noronha+Torrieri,...)

NB:AdS/CFT more general than hydrodynamics. No equilibrium assumption present, $\langle T_{\mu\nu} \rangle$ calculated from "quantum field theory". Higher order calculations (eg $\langle T_{\mu\nu} T_{\alpha\beta} ... \rangle$) possible Ab initio (unlike hydrodynamics).
NB2:all AdS/CFT calculations up til now, too idealized to be reliably compared to experiment directly. But a fast-developing field

AdS/CFT heavy ion phenomenology

Lets assume for a moment some sort of AdS/CFT correspondence is a good description of collective effects at RHIC energies. Does this have phenomenological falsifiable consequences? Beyond the true but uninteresting "no Majorana fermions and scalars seen at RHIC"

$\frac{\eta}{s} \simeq \frac{1}{4\pi}$ does not qualify, since η/s unknown within an order of magnitude (boost invariance? EoS? evolution of η?), and because corrections to $\lambda, N_c \to \infty$ deviate from this. "Drag force" on a heavy quark also beset by parameters (flow, etc). But how about both of these (hard and soft) with one set of parameters, just as in perturbative Boltzmann equation?

AdS/CFT heavy ion phenomenology

Same black holes (thermal medium) strings+branes (heavy quarks) etc, but Lagrangian more complicated

$$L_{ADS} = - \underbrace{\Lambda_{ADS} + R}_{AdS/CFT, \lambda \to \infty} + \underbrace{f(\lambda, \lambda_{GB}) \mathcal{O}(R^2, R^4)}_{SU(N_c < \infty), \lambda < \infty, Possibly\ \ \mathcal{N} < 4}$$

$$+ \underbrace{f(\phi)}_{Dilaton, breaks\ \ CFT,\ \ QFT\ \ dual\ \ \underline{unknown}}$$

Corrections due to $\lambda < \infty$ and "Gauss-Bonnet" R^2 terms to entropy:

$$s/s_{SB} = \frac{3}{4}\left(1 + \lambda_{GB} + \frac{15}{8}\frac{\zeta(3)}{\lambda^{3/2}}\right)$$

(S. S. Gubser, I. R. Klebanov and A. A. Tseytlin, NPB534, 202 (1998)).

Unsurprisingly, smaller interaction strength moves system towars s_{SB} , move is monotonic

While an explicitly conformal-braking term would be nice (and is in the works with a dilaton potential), we can at least use the physical number of degrees of freedom (the deficiency of s/s_{sb} right above T_c wrt the AdS/CFT limit) as a constraint

Correction to shear viscosity (from v_2)

$$\eta/s = \frac{1}{4\pi}\left(1 - 4\lambda_{GB} + 15\frac{\zeta(3)}{\lambda^{3/2}}\right)$$

(Buchel,Liu,Starinets, Nucl.Phys.B 707, 56 (2005))

Dependence similar to weak coupling, but not identical:

- Lower limit (quantum bound?), dependent on λ_{GB} (
- Power different wrt perturbative result $(\lambda^2 \ln \lambda)^{-1}$

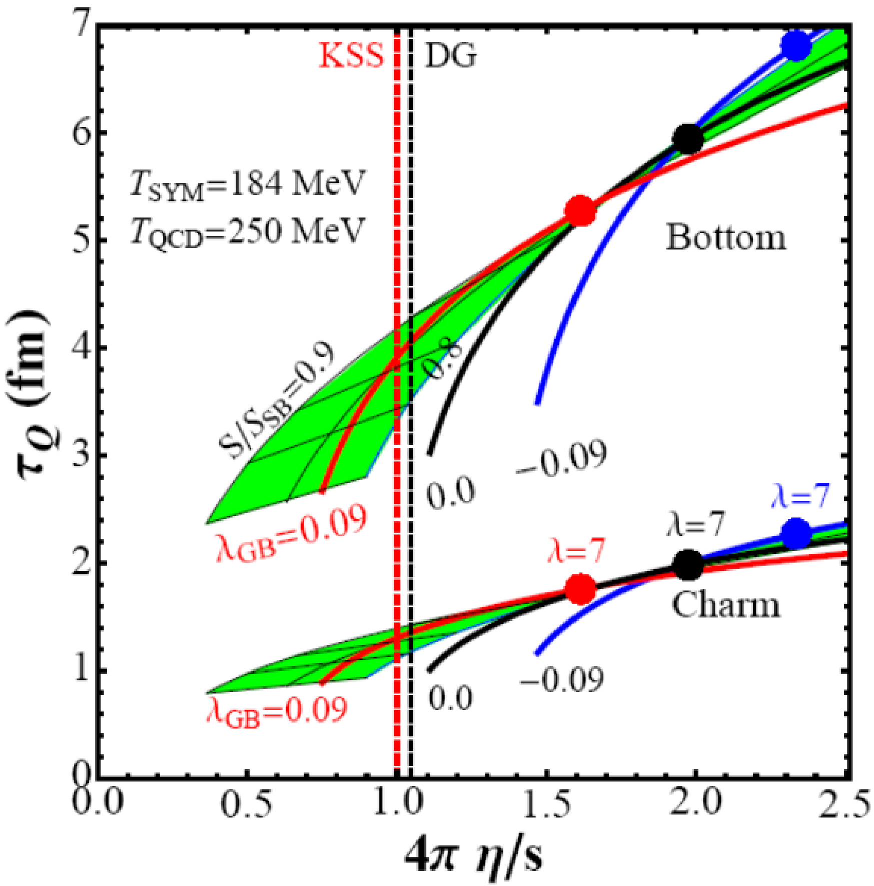
Correction to drag force on heavy quarks (on jet suppression)

$$\tau_Q^{-1} = \mu_Q \left(1 + \frac{3}{2}\lambda_{GB} + \frac{15}{16}\frac{\zeta(3)}{\lambda^{3/2}} \right)$$

(J.Noronha,2009, F.Fadafan,JHEP **0812**, 051 (2008))

big assumption: Deceleration "slow" ($\ll T?$), so one can use a formula based on "dragging the heavy quark" model at any given time
Result wholly different from perturbative calculations, possibility of limiting speed (Kharzeev)

Putting everything together...



From transport to phenomenology... background evolution

As apparent from previous discussion, (shown explicitly in Janik et al, hep-th/0606149 for Bjorken, Bhattacharyya et al 0712.2456 more generally) hydrodynamic solutions are good approximations for $\langle T_{\mu\nu} \rangle$ in a strongly coupled theories. Hence, $T(x,t)$ (necessary for microscopic calculations) can be obtained by integrating the hydrodynamic solution with the appropriate EoS and viscosity. Conformal symmetry assures that

$$s = \alpha_1 T^3 \quad , \quad \eta = \alpha_2 s \quad , \quad \zeta = 0 \quad , \quad \frac{dp}{dt} \sim T^2$$

First attempt: the 1D viscous Bjorken equation (hopefully a decent approximation for early times).

$$\frac{dT}{d\tau} = \frac{3\alpha_1 T}{\tau} + \frac{4\alpha_2}{3\tau^2}$$

From transport to phenomenology... v_2

If one neglects transverse and elliptic flow in relevant $T(\tau)$, then

$$v_2 = v_2^{ideal}\left(1 - \beta\frac{l_{mfp}}{R}\right)$$

Considering $R \sim R_0(1+\epsilon\cos\phi)$, and remembering that $v_2^{ideal} \sim \epsilon + O(\epsilon^2)$ we get to the famous

$$\frac{v_2}{\epsilon} = \frac{v_2}{\epsilon}\Big|_{ideal}\left(1 - \beta'\frac{\eta}{s}\right)$$

Fit parameters were taken from Luzum+Romatschke, 2008

From transport to phenomenology... jet suppression

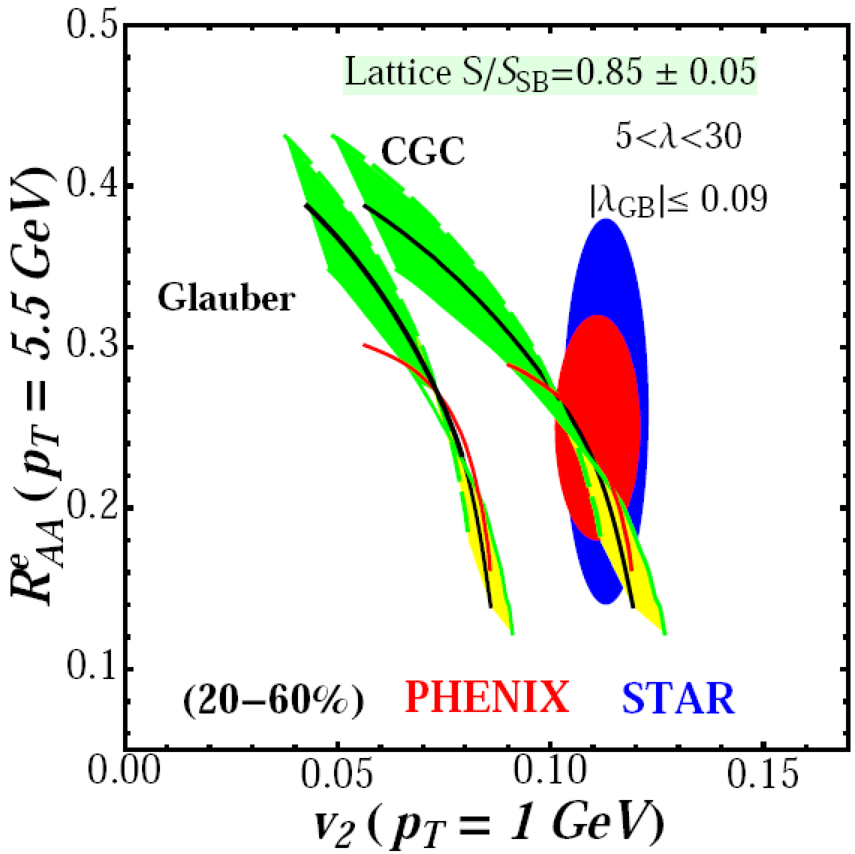
Assume deceleration slow, so drag force formula can be used. Solve

$$\int_{p_T^i}^{p_T^f} \frac{dp_T}{f(p_T)} = \int_{t_0}^{t_f} \frac{dt}{t}$$

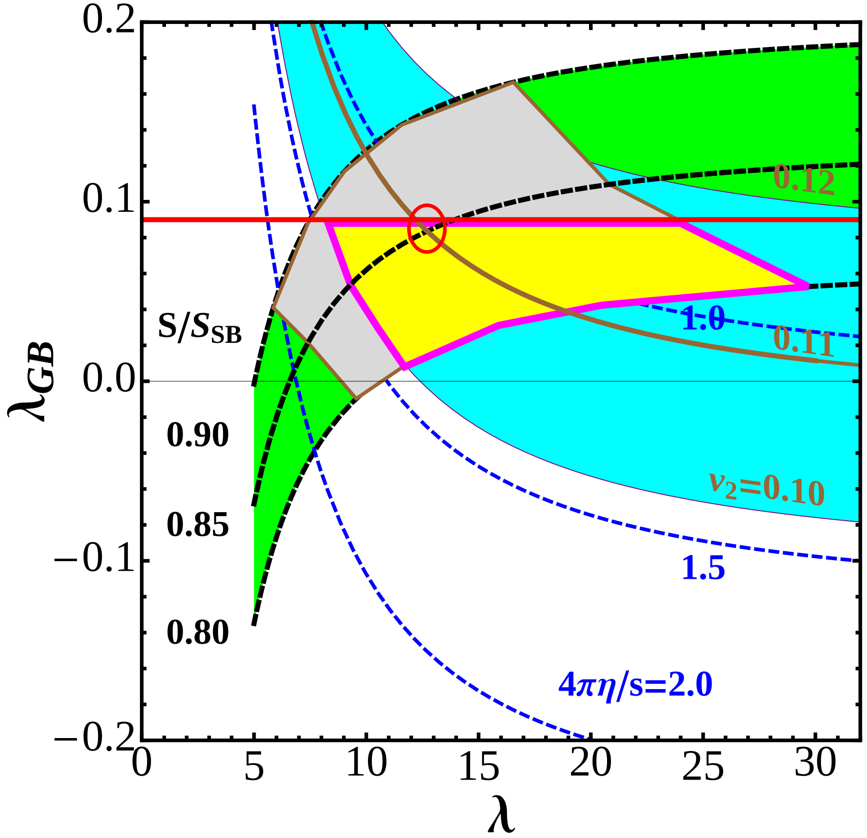
integrate over all possible paths (ie, all possible t_fs to get $\left\langle p_T^i \right\rangle (p_T^f)$, the initial momentum required to get a final momentum p_T^f .R_{AA} will then be given simply by

$$R_{AA} = \left(E\frac{dN}{d^3p} \right)_{p=p_T^i(p_T^f)} / \left(E\frac{dN}{d^3p} \right)_{p_T^f}$$

Putting everything (and data) together...



Inverting the constraints...



To do...

Theory

- Break conformal symmetry explicitly (dilaton potential)
- Use a realistic hydrodynamic code, rather than Bjorken (coming soon!)

Phenomenology A detailed comparison between weakly coupled (transport) with strongly coupled (Hydro+Drag force, with appropriate transport coefficients). Is a signature capable of distinguishing $\lambda \gg 1$ from $\lambda \ll 1$ on the horizon?

- Transport
- Fokker-Plank equation (Moore-Teaney, Phys.Rev.C71:064904,2005)

Spare slides