

Spin polarization phenomena in dense neutron matter at a strong magnetic field



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Introduction

Neutron stars observed in nature are magnetized objects with the magnetic field strength at the surface in the range 10^9 - 10^{13} G.

R.C. Duncan, and C. Thompson (1992): even more strongly magnetized objects, called **magnetars**, can exist in the universe **with the field strength about 10^{14} - 10^{15} G at the surface**. Golden candidates for magnetars: soft gamma-ray repeaters and anomalous X-ray pulsars. Nowadays it is believed that magnetars constitute about 10% of the whole population of neutron stars.

In the interior of a magnetar the magnetic field strength may be even larger, reaching the values about 10^{18} G.

Possible mechanisms to generate such strong magnetic fields:

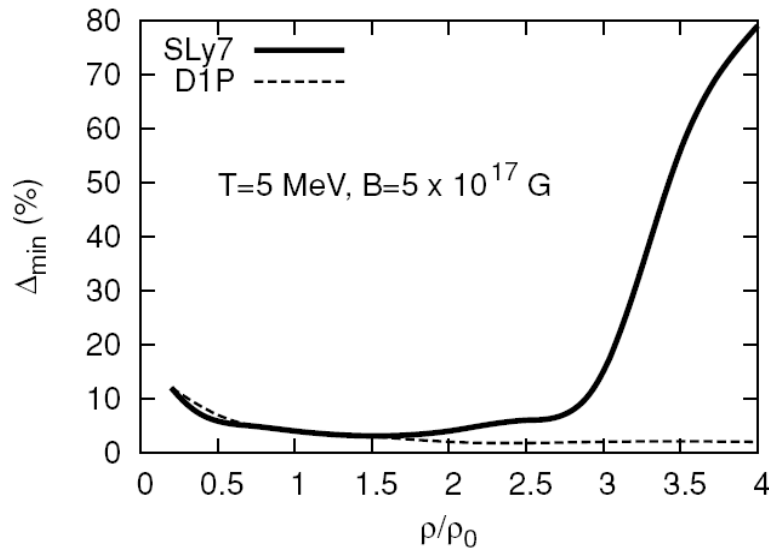
- turbulent dynamo amplification mechanism in rapidly rotating neutron stars - C. Thompson and R.C. Duncan, ApJ 473 (1996) 322;
- the possibility of spontaneous spin ordering in the dense quark core of a neutron star - T. Tatsumi, Phys. Lett. B489 (2000) 280.

Introduction

The issue of interest: the behavior of a neutron star matter in a strong magnetic field up to 10^{18} G, considered as created externally.

Approximation for neutron star matter: pure neutron matter

$$\Delta = \frac{\rho_{\uparrow} - \rho_{\downarrow}}{\rho}$$



M. A. Perez-Garcia
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Two possible scenarios for the behavior of spin polarization of dense neutron matter at a strong magnetic field:

- a ferromagnetic transition to a totally spin polarized state occurs at high densities (Skyrme effective NN interaction);
- a ferromagnetic transition is excluded at all relevant densities and the spin polarization remains quite low even in the high density region (Gogny effective D1P NN interaction).

Introduction

Which scenario is realized crucially depends on whether:

- spontaneous spin polarization (in the absence of a magnetic field) is developed in neutron matter at several times nuclear matter saturation density - Skyrme interaction,
- or the appearance of a spontaneous polarization is forbidden at the relevant densities (or delayed to much higher densities) - Gogny D1P interaction.

The issue of spontaneous appearance of spin polarized states in neutron matter is a controversial one:

- **Skyrme effective interaction:** the spontaneous spin instability occurs in neutron matter at densities in the range from ρ_0 to $4\rho_0$ for different parametrizations of the NN potential – A. A. Isayev, and J. Yang, PRC 69, 025801 (2004); A. A. Isayev, PRC 74, 057301 (2006).
- **Gogny effective interaction:** a ferromagnetic transition in neutron matter occurs at the density larger than $7\rho_0$ for the D1P parametrization and is not allowed for D1, D1S parametrizations - D. Lopez-Val, A. Rios, A. Polls, and I. Vidana, PRC 74, 068801 (2006).
- **Realistic NN interactions:** no sign of spontaneous spin instability at any isospin asymmetry has been found so far up to densities well above ρ_0 – F. Sammarruca, and P. G. Krastev, PRC 75, 034315 (2007).

Introduction

Main objective: to study thermodynamic properties of spin polarized neutron matter at a strong magnetic field in the model with the Skyrme effective forces.

Theoretical framework: a Fermi liquid description of neutron matter

A. I. Akhiezer, A. A. Isayev, S. V. Peletminsky, A. P. Rekalov, and A. A. Yatsenko, JETP 85, 1 (1997).

A.A. Isayev, and J. Yang, in *Progress in Ferromagnetism Research*, edited by V.N. Murray, Nova Science Publishers, New York, 2006, p. 325.

The self-consistent equations for the spin order parameter and chemical potential of n



A few branches of solutions for the spin order parameter



Thermodynamic analysis of different branches of solutions and conclusion about the possibility of a new scenario of evolution of the spin structure in the interior of a magnetar

Basics of Formalism

The normal (nonsuperfluid) states of neutron matter are described by the normal distribution function of neutrons

$$f_{\kappa_1\kappa_2} = \text{Tr } \rho a_{\kappa_2}^+ a_{\kappa_1}, \quad \kappa \equiv (\vec{p}, \sigma)$$

The matrix self-consistent equation for f in the state of thermodynamic equilibrium

$$f = \left\{ \exp(Y_0 \varepsilon + Y_4) + 1 \right\}^{-1} \equiv \left\{ \exp Y_0 \xi + 1 \right\}^{-1}$$

The single particle energy

$$\varepsilon_{\kappa_1\kappa_2}(f) = \frac{\partial E(f)}{\partial f_{\kappa_2\kappa_1}}$$

and

$$Y_{4\kappa_1\kappa_2} = Y_4 \delta_{\kappa_1\kappa_2}, \quad Y_0 = \frac{1}{T}, \quad Y_4 = -\frac{\mu_0}{T}$$

Basics of Formalism

Structure of the normal distribution function and single particle energy for ordering || \mathbf{H}

$$f(\vec{p}) = f_0(\vec{p})\sigma_0 + f_3(\vec{p})\sigma_3$$

$$\varepsilon(\vec{p}) = \varepsilon_0(\vec{p})\sigma_0 + \varepsilon_3(\vec{p})\sigma_3$$

Normalization conditions for the distribution functions

$$\frac{2}{V} \sum_{\vec{p}} f_0(\vec{p}) = \rho_{\uparrow} + \rho_{\downarrow} \equiv \rho \quad \leftarrow \text{total density}$$

$$\frac{2}{V} \sum_{\vec{p}} f_3(\vec{p}) = \rho_{\uparrow} - \rho_{\downarrow} \equiv \Delta\rho \quad \leftarrow \text{neutron spin order parameter}$$

Magnetization of the system

$$\mathbf{M} = \mu_n \Delta\rho, \quad \mu_n = -1.91\mu_N \approx -6.03 \cdot 10^{-18} \text{ MeV/G}$$

Internal magnetic field $\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M}$

Approximation: $\mathbf{B} \approx \mathbf{H}$ (confirmed numerically for all relevant ρ and \mathbf{H})

Basics of Formalism

The energy functional of the system

$$E(f) = E_0(f, H) + E_{\text{int}}(f) + E_{\text{field}}$$

$$E_0(f, H) = 2 \sum_{\vec{p}} \varepsilon_0(\vec{p}) f_0(\vec{p}) - 2\mu_n H \sum_{\vec{p}} f_3(\vec{p}), \quad \varepsilon_0(\vec{p}) = \frac{\vec{p}^2}{2m_0}$$

$$E_{\text{int}}(f) = 2 \sum_{\vec{p}} \{ \tilde{\varepsilon}_0(\vec{p}) f_0(\vec{p}) + \tilde{\varepsilon}_3(\vec{p}) f_3(\vec{p}) \}, \quad E_{\text{field}} = \frac{H^2}{8\pi} V$$

Fermi liquid corrections to the free single particle spectrum

$$\tilde{\varepsilon}_0(\vec{p}) = \frac{1}{2V} \sum_{\vec{q}} U_0^n(\vec{k}) f_0(\vec{q}), \quad \vec{k} = \frac{\vec{p} - \vec{q}}{2}$$

$$\tilde{\varepsilon}_3(\vec{p}) = \frac{1}{2V} \sum_{\vec{q}} U_1^n(\vec{k}) f_3(\vec{q})$$

FL amplitudes U_0 , U_1 describe density and spin correlations in neutron matter

Basics of Formalism

Self-consistent equations

$$\begin{cases} \xi_0(\vec{p}) = \varepsilon_0(\vec{p}) - \mu_0 + \frac{1}{2V} \sum_{\vec{q}} U_0^n(\vec{k}) f_0(\vec{q}) \\ \xi_3(\vec{p}) = -\mu_n H + \frac{1}{2V} \sum_{\vec{q}} U_1^n(\vec{k}) f_3(\vec{q}) \end{cases}$$
$$f_0 = \frac{1}{2} \{n(\omega_+) + n(\omega_-)\}, \quad f_3 = \frac{1}{2} \{n(\omega_+) - n(\omega_-)\}$$
$$\omega_{\pm} = \xi_0 \pm \xi_3$$

Set of integral equations for the components of the single particle energy to be solved self-consistently.

Numerical procedure: iterations on the Gaussian grid in momentum space until convergence with required accuracy is achieved.

Basics of Formalism

Amplitude of NN interaction for Skyrme effective forces

$$v(\vec{p}, \vec{p}') = t_0(1 + x_0 P_\sigma) + \frac{1}{6} t_3(1 + x_3 P_\sigma) \rho^\beta + \frac{1}{2\hbar^2} t_1(1 + x_1 P_\sigma)(\vec{p}^2 + \vec{p}'^2) \\ + \frac{t_2}{\hbar^2}(1 + x_2 P_\sigma) \vec{p} \vec{p}', \quad P_\sigma = \frac{1}{2} (1 + \vec{\sigma}_1 \vec{\sigma}_2)$$

In numerical calculations: Skyrme SLy4 and SLy7 effective forces – were developed to reproduce EOS of neutron matter obtained in microscopic calculations.

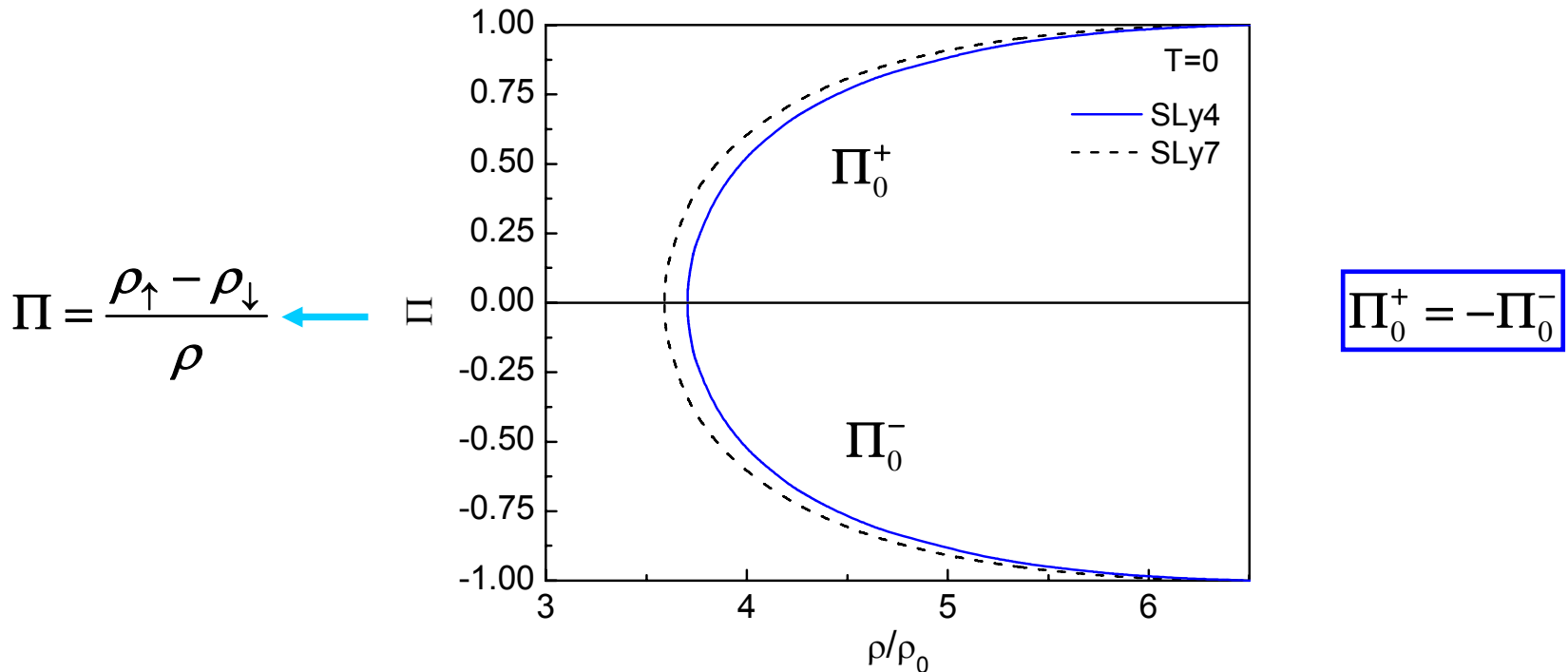
Magnetic fields up to the values allowed by a scalar virial theorem.

For a neutron star with mass M and radius R :

$$\text{Magnetic field energy } E_H \sim \frac{4\pi R^3}{3} \frac{H^2}{8\pi} \quad \text{Gravitational binding energy } E_G \sim \frac{GM^2}{R}$$

$$E_H = E_G \quad \rightarrow \quad H_{\max} \sim \sqrt{6G} \frac{M}{R^2} \quad \boxed{M = 1.5M_\odot, R = 10^{-5} R_\odot} \quad \rightarrow \quad \boxed{H_{\max} \sim 10^{18} \text{ G}}$$

Spontaneous Spin Polarization (H=0)

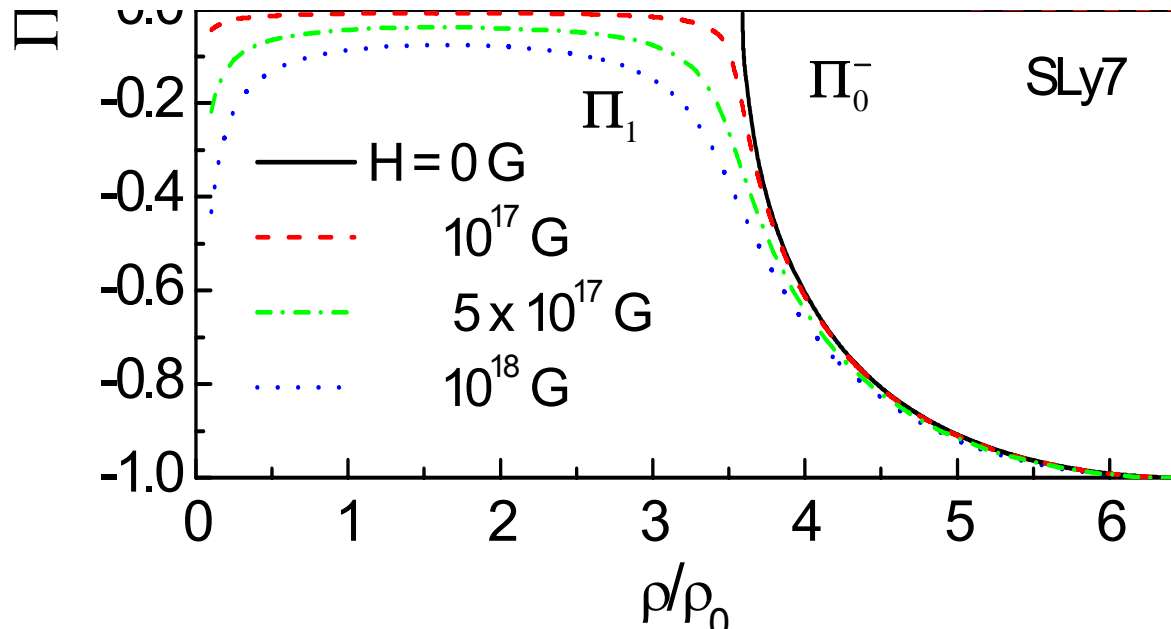


Neutron spin polarization parameter as a function of density at vanishing T and H

The spontaneous polarization develops at $\rho = 3.70 \rho_0$ for the SLy4 interaction ($\rho_0 = 0.16 \text{ fm}^{-3}$) and at $\rho = 3.70 \rho_0$ ($\rho_0 = 0.158 \text{ fm}^{-3}$) for the SLy7 interaction. There are two branches of spontaneous polarization, Π_0^- , Π_0^+ .

Spin polarization in neutron matter at strong H

Modification of the lower branch Π_0^- of spontaneous polarization by H

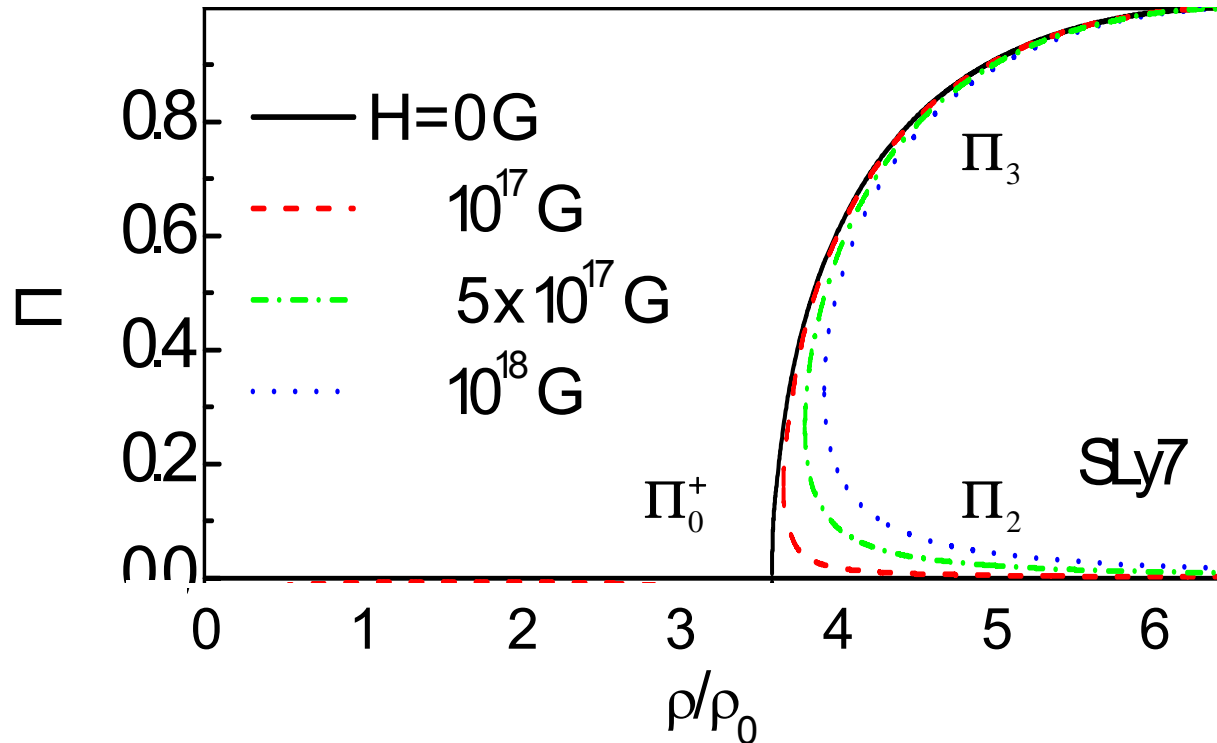


Three characteristic density domains for the modified lower branch Π_1 :

- At low densities $\rho \leq 0.5 \rho_0$, the magnitude of the spin polarization parameter increases with decreasing density.
- At intermediate densities $0.5 \rho_0 \leq \rho \leq 3 \rho_0$, there is a plateau in the $\Pi_1(\rho)$ dependence, whose characteristic value depends on H , e.g., $\Pi_1 \approx -0.08$ at $H = 10^{18}$ G.
- At densities $\rho \geq 3 \rho_0$, the magnitude of the spin polarization parameter increases with density, and neutrons become totally polarized at $\rho \approx 6 \rho_0$

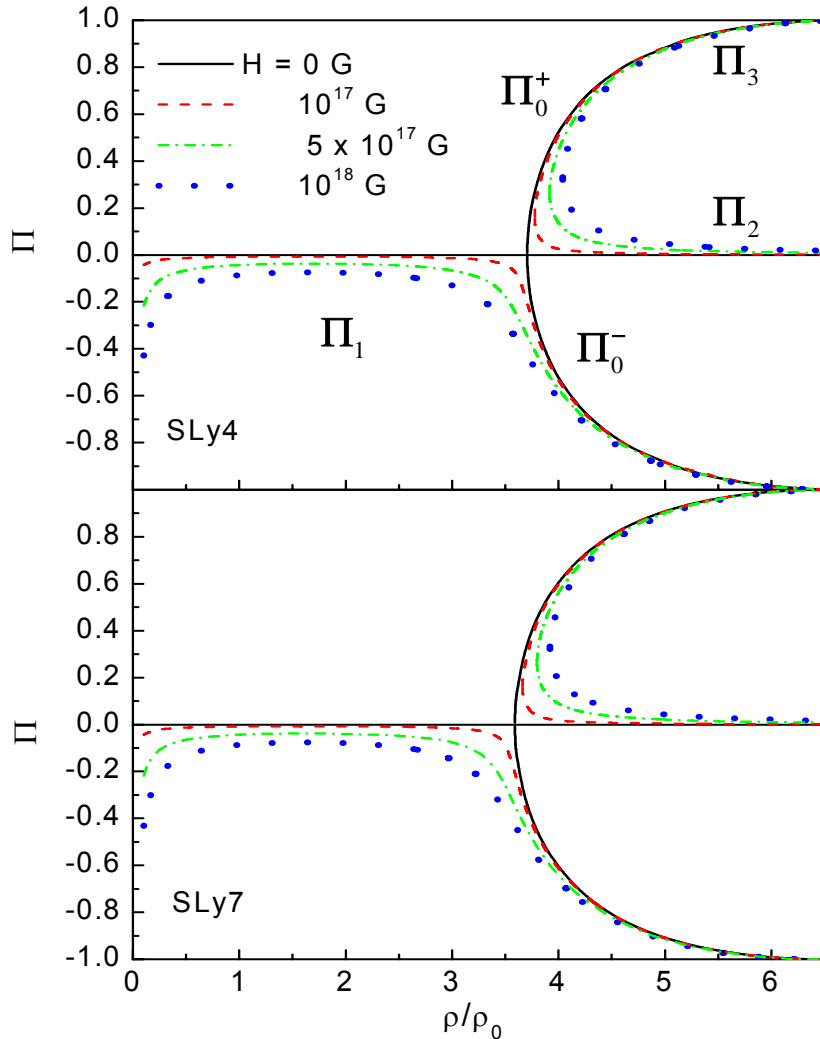
Spin polarization in neutron matter at strong H

Modification of the upper branch Π_0^+ of spontaneous polarization by H



The upper branch of spontaneous polarization Π_0^+ at nonzero magnetic field turns into two branches Π_2 , Π_3 . For Π_2 , the spin polarization parameter decreases with density and tends to zero value while for Π_3 it increases with density and is saturated. These branches appear step-wise at the same threshold density ρ_{th} , e.g. for SLy7 interaction, $\rho_{th}=3.8 \rho_0$ at $H=5 \cdot 10^{17}$ G, and $\rho_{th}=3.92 \rho_0$ at $H=10^{18}$ G.

Spin polarization in neutron matter at strong H



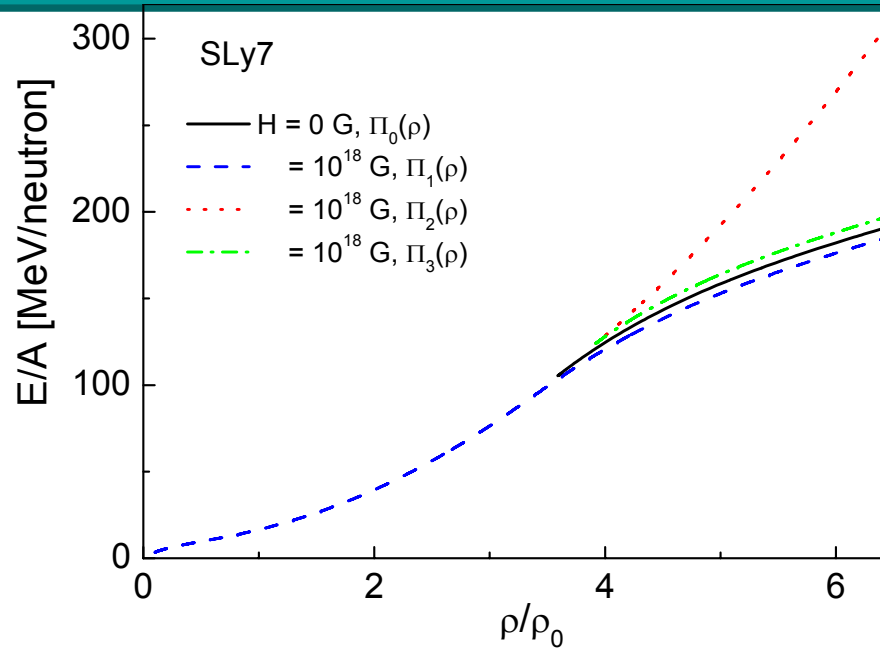
At densities $\rho > \rho_{th}$, there are three branches of solutions:

- Π_1 with the negative spin polarization,
- Π_2 and Π_3 with the positive polarization.

Peculiarities:

- The magnetic field, due to $\mu_n < 0$, tends to orient the neutron spins oppositely to \mathbf{H} . As a result, $\Pi_2, \Pi_3 < \Pi_0^+$, $|\Pi_1| > |\Pi_0^-|$.
- The impact of even such a strong magnetic field as $H = 10^{17}$ G is small: The spin polarization parameter for all three branches $\Pi_1 - \Pi_3$ is either close to zero, or close to its value in the state with spontaneous polarization, which is governed by the spin-dependent medium correlations.

Analysis of Thermodynamic Stability

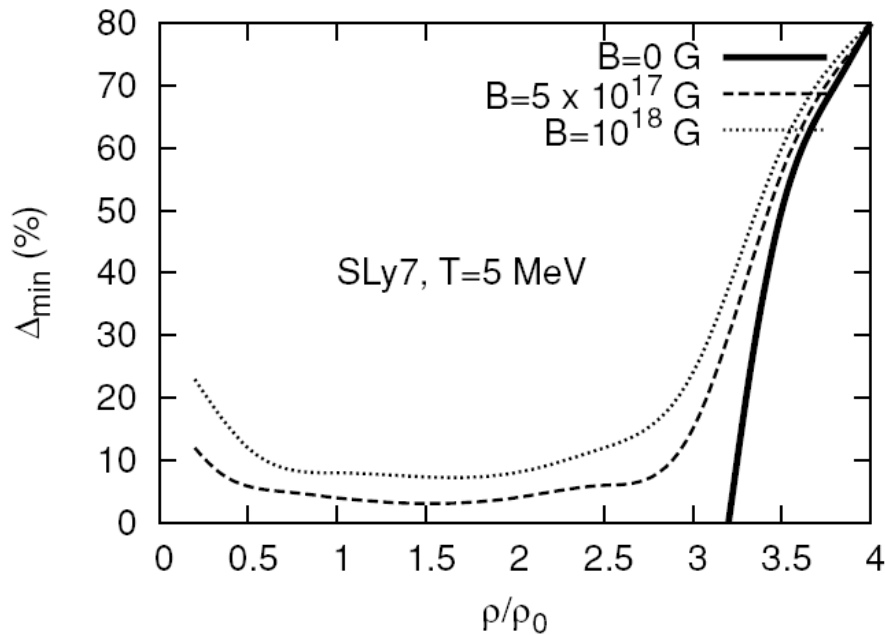


Results:

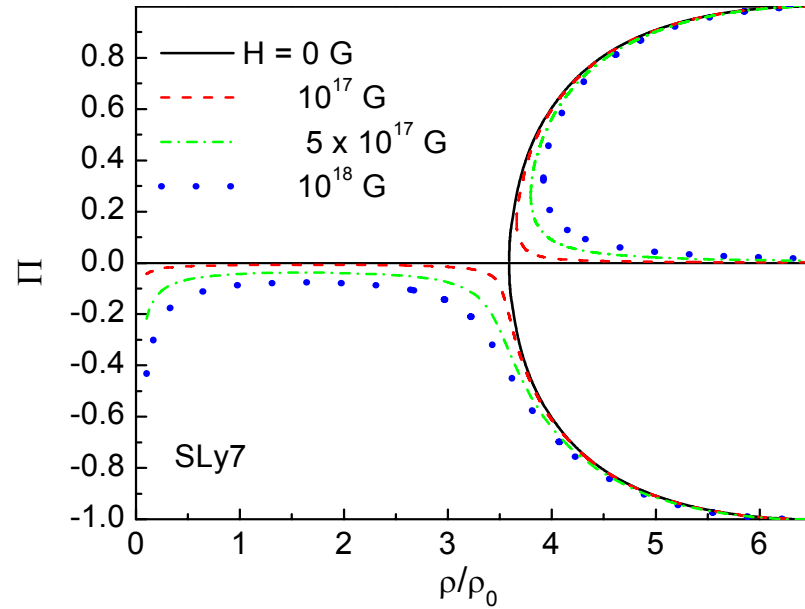
- The state with the majority of neutron spins oriented oppositely to \mathbf{H} [the branch $\Pi_1(\rho)$] is thermodynamically preferable.
- The state, corresponding to the branch $\Pi_3(\rho) > 0$, has the energy very close to that of the thermodynamically stable state.

Hence, despite the presence of a strong magnetic field $H \sim 10^{18} \text{G}$, the state with the majority of neutron spins directed along H can be realized as a metastable state in the dense core of a magnetar in the model with the Skyrme NN interaction. In this scenario, since such states exist only at $\rho > \rho_{\text{th}}$, under decreasing density (going from the interior to the outer regions of a magnetar) a metastable state with the positive spin polarization at the threshold density ρ_{th} changes into a thermodynamically stable state with the negative spin polarization.

Comparison with previous results



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present research

• Only one branch of solutions for Π was found in the model with the Skyrme force



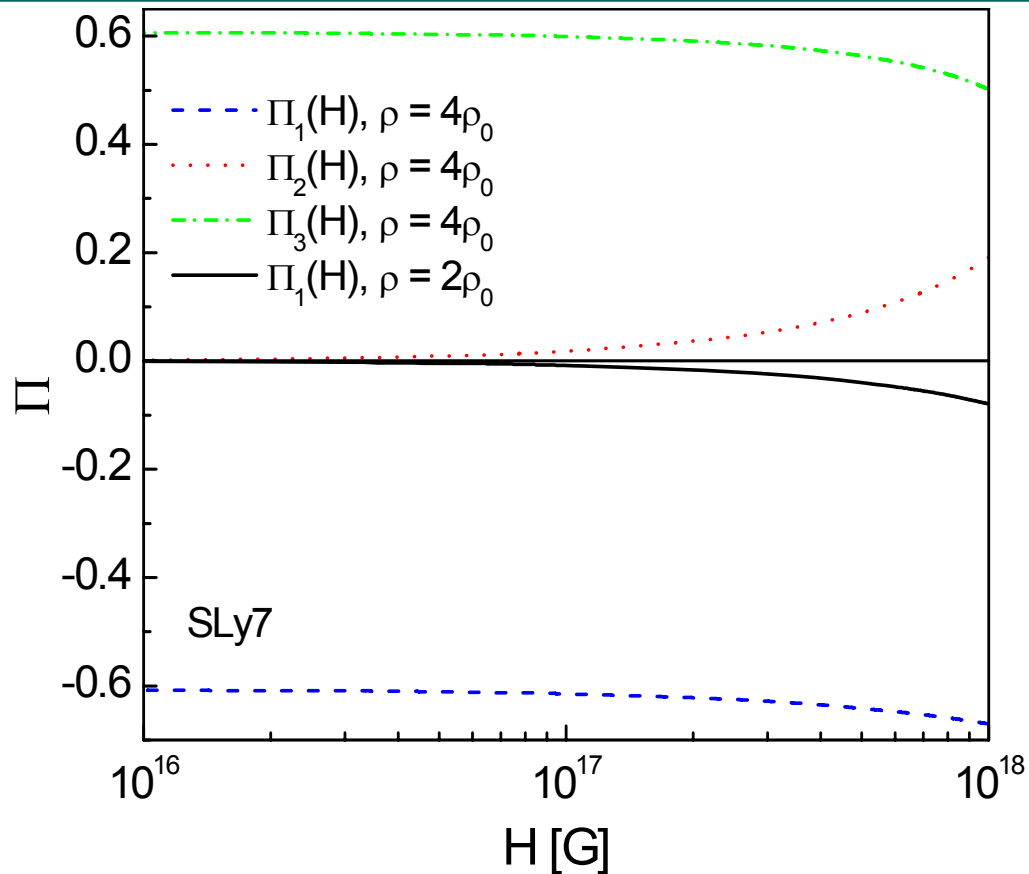
• In the Skyrme model, in general, there are 3 different branches of solutions for Π at nonvanishing H .

• The only branch is characterized by the positive spin polarization $\Pi > 0$



• For a thermodynamically stable branch the spin polarization is negative, $\Pi < 0$

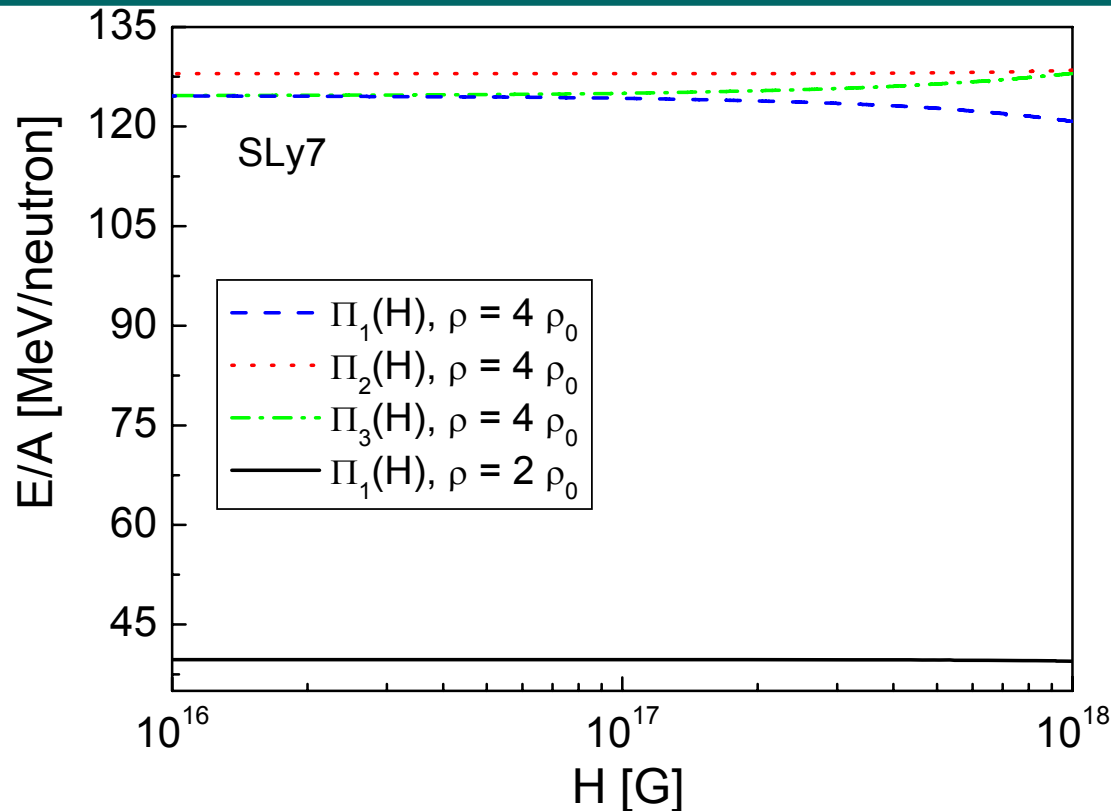
Spin polarization vs. magnetic field strength



Results:

- up to the field strengths $H=10^{17}$ G, the influence of the magnetic field is rather marginal;
- for the branches $\Pi_1(H)$ and $\Pi_2(H)$, the magnitude of the spin polarization parameter increases with the field strength while for the $\Pi_3(H)$ it decreases.

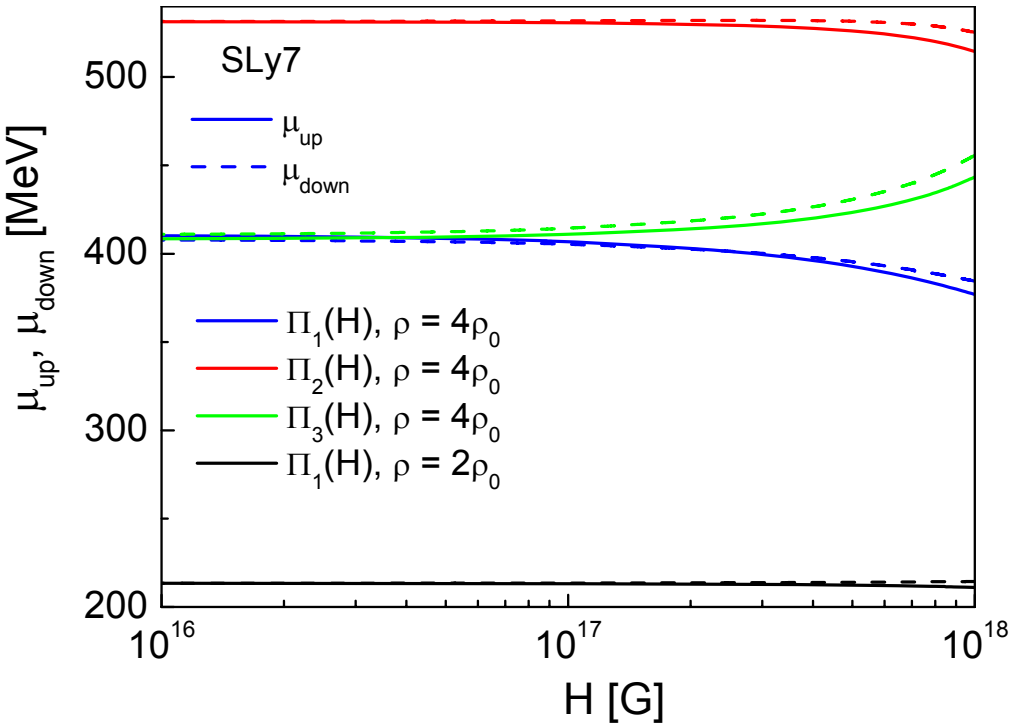
Energetics of neutron matter vs. magnetic field



Results:

- the state with $\Pi < 0$ [branch $\Pi_1(H)$] becomes more preferable with increasing H although the total effect of changing H by two orders of magnitude on the energy corresponding to all three branches Π_1 - Π_3 remains small.
- increase of the density by a factor of two leads to the increase in the energy per neutron roughly by a factor of three and hence the medium correlations play more important role in building the energetics of the system than the impact of a strong H .

Chemical potentials vs. magnetic field



$$\mu_{\pm} = \mu_{\text{eff}} \pm \left(\mu_n H - \frac{a_n \Delta \rho}{4} - \frac{b_n}{16} \langle q^2 \rangle_3 \right)$$

Chemical potentials of spin-up (solid) and spin-down (dash) neutrons vs. H:

- there is a small effect of a magnetic field up to the field strengths 10^{17} G;
- $\mu_{\uparrow}, \mu_{\downarrow}$ for branches Π_1, Π_2 decrease with H, for Π_3 increase with H;
- from comparison of $\mu_{\uparrow}, \mu_{\downarrow}$ for branch Π_1 at $\rho=2\rho_0$ and $\rho=4\rho_0$: in-medium effects strongly increase the effective chemical potentials

Conclusions

Spin polarized states in neutron matter at a strong magnetic field up to 10^{18} G allowed by scalar virial theorem have been considered in the model with the Skyrme effective NN interaction (SLy4, SLy7 forces). It has been shown that:

- A thermodynamically stable branch of solutions of the self-consistency eqs. for the spin polarization parameter Π corresponds to the case when the majority of neutron spins are oriented oppositely to the direction of the magnetic field (negative spin polarization).
- Beginning from some threshold density $\rho_{\text{th}}(H)$ ($\sim 4\rho_0$) the self-consistent equations have also two other branches (upper and lower) of solutions for the spin polarization parameter corresponding to the positive spin polarization.
- A free energy corresponding to the upper branch turns out to be very close to that of a thermodynamically preferable branch with $\Pi < 0$. As a consequence, at a strong magnetic field, the state with the majority of spins aligned along H can be realized as a metastable state in dense neutron matter which under decreasing density (going from the interior to the outer regions of a magnetar) at the threshold density ρ_{th} changes into a thermodynamically stable state with the majority of spins aligned oppositely to H .
- The calculations of the neutron spin polarization parameter, energy per neutron and chemical potentials of spin-up and spin-down neutrons show that the influence of the magnetic field remains small up to the field strengths 10^{17} G.