Coherent charge transport in strongly correlated systems and quasiparticle pair production

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It is known that fermion pair creation appears in a large number of physical situations described in condensed matter, atomic, nuclear, elementary particle physics, astrophysics, and cosmology. Therefore, the problem of pair production in electric fields has been the subject of considerable theoretical interest. To date, an analytic formalism that successfully addresses the general problem of fields which vary arbitrarily in both time and space has not been developed.

The goal of my talk is to consider the charge transport in strongly correlated systems by means of Dyson equation for a matrix Green function. With the goal, the secondary quantized wave function of fermion multiple systems will be found in terms of one-particle fermion creation (annihilation) operators and two-particle creation (annihilation) operators, and the Green function method developed will be applied for the quantum field description of the problem of pair creation.

1. Dirac particle in a homogeneous electric field

Dirac equation describes a classical fermion field. However the Dirac operator has unphysical states that leads to Klein paradox in a problem of electron scattering on a potential barrier.

The Dirac equation describes a motion of an electron, and its Dirac conjugation describes a positron motion. Therefore that fact is surprising, that the states belonging to the energy gap of the Dirac operator, describe a fermion pair arising in a homogeneous electric field rotating in plane.

The Dirac equation for an amplitude of electron state $|\phi_0>$ in an electric field $\vec{E}(t)$ is written through the prolonged derivative $\vec{P}-ie\vec{A}$ on space coordinates:

$$\imath\frac{\partial}{\partial t}|\phi_0> = \left[\left(\widehat{\vec{P}}-ie\vec{A}(t)\right)\cdot\vec{\alpha}+m_e\beta\right]|\phi_0>, \qquad (1.1)$$

where $\vec{A}(t)=-\int\vec{E}(t)dt$ is a vector-potential. We shall look for a solution of the equation (1.1) as

$$<\vec{r},t|\phi_0> = \exp(\imath\vec{p}\cdot\vec{r})<t|\phi_0>. \qquad (1.2)$$

Substituting the expression (1.2) in the equation (1.1), we obtain the following equation for the bispinor $<t|\phi_0>\equiv\phi_0(t)$:

$$\imath\frac{\partial}{\partial t}\phi_0(t)=\left(\vec{\pi}(t)\cdot\vec{\alpha}+m_e\beta\right)\phi_0(t), \qquad (1.3)$$

where $\vec{\pi}(t)=\vec{p}-\vec{A}(t)=\vec{p}+\int\vec{E}(t)dt$. Symmetry of the problem (1.3) is described by the group SO (4) [Perelomov].

2. Secondary quantized problem of pair production

A secondary quantized fermion field ψ_{in} is represented as set of electrons and positrons, described by the complex spinor

$$\psi_{\text{in}} = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \sqrt{\frac{m}{k_0}} \sum_{\beta} [b_\beta(k) u_\beta(k) e^{-ik\cdot x} + d^\dagger_\beta(\tilde{k}) v_\beta(\tilde{k}) e^{ik\cdot x}], \qquad (2.1)$$

which real components u_β are electronic ones, and imaginary components v_β are positronic ones [Gribov]. The interaction Hamiltonian for fermions is determined by

$$H_I = -eA_i \int d^3x \bar{\psi}_{in}(x) \gamma_i \psi_{in} \qquad (2.2)$$

Now, we rewrite the interaction Hamiltonian for fermions in an electric field by adopting the gauge

$$A_0 = 0, \quad A_i = -\int_{-\infty}^{t} E_i(t)\, dt \qquad (2.3)$$

Then the interaction Hamiltonian is given by [Seger, Balantekin]

$$H = \int d^3k \left\{ \left(2k_0 - 2e\frac{A_i k_i}{k_0} \right) J_0(k) - e\frac{\mu_i A_i k_i}{k_0} \left[J_+^{(i)}(k) + J_-^{(i)}(k) \right] \right\} \tag{2.4}$$

where incoming Dirac field ψ_{in} is that of free fermions,

$$J_+^{(i)} = \frac{m}{\mu_i} \sum_{\alpha\beta} b_\alpha^\dagger(k) d_\beta^\dagger(\tilde{k}) \bar{u}_\alpha(k) \gamma_i v_\beta(\tilde{k}), \quad i = 1,2,$$

$$J_-^{(i)} = [J_+^{(i)}]^\dagger, \quad i = 1,2,$$

$$J_0 = \frac{1}{2} \sum_\alpha [b_\alpha^\dagger(k) b_\alpha(k) - d_\alpha(\tilde{k}) d_\alpha^\dagger(\tilde{k})]. \qquad Q = \sum_{\alpha\beta} [b_\alpha^\dagger b_\beta \bar{u}_\alpha \gamma_3 \gamma_5 u_\beta + d_\alpha d_\beta^\dagger \bar{v}_\alpha \gamma_3 \gamma_5 v_\beta], \quad \mu_i = \sqrt{k_0^2 - k_i^2}$$

With additional operator Q the operators $J_\pm^i$, *i=1, 2* produce the algebra of group SO (4) which is locally isomorphic to the group SU (2) x SU (2).

Now, we find representation on which the states described by the Hamiltonian for the problem of pair production are transformed.

We can define an operator of electron creation $\widehat{\phi}_1^{\,+}$ as quantized positively frequency part ϕ_1^{+} of field function ϕ_1, and an operator of positron creation $\widehat{\phi^{\dagger}}_1^{\,+}$ as quantized positively frequency part $\phi^{\dagger}{}_1^{+}$ of Hermitian conjugate field function $\phi^{\dagger}{}_1$. Accordingly, the operator of electron annihilation $\widehat{\phi^{\dagger}}_1^{\,-}$ is defined as quantized negatively frequency part $\phi^{\dagger}{}_1^{-}$ of Hermitian conjugate field function $\phi^{\dagger}{}_1$, and the operator of positron annihilation $\hat{\phi}_1^{\,-}$ as quantized negatively frequency part ϕ_1^{-} of field function ϕ_1.

Now we can define annihilation operators $\left(\widehat{\Phi}^{\dagger}{}_{pair}\right)^{-}$ and creation operators $\left(\widehat{\Phi}_{pair}\right)^{+}$ of fermion pairs as

$$\left(\widehat{\Phi}^{\dagger}{}_{pair}\right)^{-} = \Phi_{-} = \hat{\phi}_1^{\,-}\widehat{\phi^{\dagger}}_1^{\,-}, \qquad (2.4)$$

$$\left(\widehat{\Phi}_{pair}\right)^{+} = \Phi_{+} = \widehat{\phi^{\dagger}}_1^{\,+}\widehat{\phi}_1^{\,+}. \qquad (2.5)$$

and an operator $$\left(\widehat{\Phi}^{\dagger}{}_{pair}\right)^{0} = \Phi_0 = \frac{1}{2}\left(\widehat{\phi}_1^{\,+}\widehat{\phi^{\dagger}}_1^{\,-} - \hat{\phi}_1^{\,-}\widehat{\phi^{\dagger}}_1^{\,+}\right). \qquad (2.6)$$

Substituting into invariant Casimir operator C_2 for algebra SU (2) $$C_2 = \frac{1}{2}(\Phi_{+}\Phi_{-} + \Phi_{-}\Phi_{+}) + \Phi_0^2 \qquad (2.7)$$

the explicit expressions for $\Phi_{\pm}, \Phi_0$ (2.5) – (2.7), we find the Casimir operator as

$$C_2 = \frac{3}{4}\left(1 - \left(\widehat{\phi}_1^{\,+}\widehat{\phi^{\dagger}}_1^{\,-} - \widehat{\phi^{\dagger}}_1^{\,+}\hat{\phi}_1^{\,-}\right)^2\right). \qquad (2.8)$$

One gets a wave function $\Psi(\vec{r}_1,\vec{r}_2)$ of fermion pair with additional coupled electron by an action of the operator $\widehat{\phi}_1^{\,+}$ and $\left(\widehat{\Phi}_{pair}\right)^+$ on a vacuum vector $|0\rangle$ as

$$\Psi(\vec{r}_1,\vec{r}_2)=\left\langle \vec{r}_1 |\,\hat{\Psi}\,| \vec{r}_2 \right\rangle=\left\langle \vec{r}_1 |\,\widehat{\phi}_1^{\,+}\,|0\right\rangle\left\langle 0 |\left(\widehat{\Phi}_{pair}\right)^+ | \vec{r}_2 \right\rangle, \qquad (2.9)$$

where $\vec{r}_1$ is a radius-vector of electron with spin "up", $\vec{r}_2$ is a radius-vector of electron with spin "down". Since by virtue of state orthogonality it is possible to add the projection operator $|0><0|$ in calculations up to $\hat{I}$, the expression (2.9) can be transformed to the form

$$|\Psi\rangle=\hat{\Psi}\,|0\rangle=\widehat{\phi}_1^{\,+}\hat{I}\left(\widehat{\Phi}_{pair}\right)^+|0\rangle. \qquad (2.10)$$

Neglecting correlations, the vacuum vector $|0\rangle$ can be presented as a product of vacuum vectors $|0\uparrow\rangle$ and $|0\downarrow\rangle$ for states with spin "up" and "down". Hence, the expression (2.10) can be rewritten as

$$|\Psi\rangle=\hat{\Psi}\,|0\rangle=\widehat{\phi}_1^{\,+}|0\uparrow\rangle\left(\widehat{\Phi}_{pair}\right)^+|0\downarrow\rangle\equiv|1,0\rangle|1,1\rangle, \qquad (2.11)$$

where $|1,0\rangle$ is a state with one electron, $|1,1\rangle$ is a state with one electron and one positron. Since, as shown above, ket-vectors $|1,0\rangle$ and $|1,1\rangle$ are transformed on a representation of the symmetry group SU(2), the wave function $|\Psi\rangle$ is transformed on representation of the symmetry group SO(4).

Let us evaluate a value which is accepted by Casimir operator C_4 of group SO (4) on the vector $|1,0\rangle|1,1\rangle$ (2.8) of the Fock space:

$$C_4(|1,0\rangle|1,1\rangle) = \left(C_2\,|1,0\rangle\right)|1,1\rangle + |1,0\rangle\left(C_2\,|1,1\rangle\right). \qquad (2.12)$$

Values of the Casimir operator C_4 (2.9) are eigenvalues of the operator of squared angular momentum $\hat{j}^2$ of the state $|1,0\rangle|1,1\rangle$ describing the system from one electron and one pair of particle – antiparticle. We see that operators $\widehat{\phi_1}^{+}\widehat{\phi^{\dagger}}_1^{-}$ and $\widehat{\phi^{\dagger}}_1^{+}\hat{\phi}_1^{-}$ are operators of occupation numbers for fermions $\hat{n}_-$ and antifermions $\hat{n}_+$:

$$\hat{n}_- = \widehat{\phi_1}^{+}\widehat{\phi^{\dagger}}_1^{-}, \quad \hat{n}_+ = \widehat{\phi^{\dagger}}_1^{+}\hat{\phi}_1^{-}. \qquad (2.13)$$

Substituting the expressions (2.8) and (2.13) in the formula (2.12), we get $C_4 = \frac{3}{4}$, as for the state $|1,1\rangle$ we have

$$C_2\,|1,1\rangle = \frac{3}{4}[1-(\hat{n}_-\,|1,1\rangle - \hat{n}_+\,|1,1\rangle)] = \frac{3}{4},$$

and for the state $|1,0\rangle$ $C_2 = 0$ owing to identity

$$C_2\,|1,0\rangle = \frac{3}{4}(1-\hat{n}_-\,|1,0\rangle + \hat{n}_+\,|1,0\rangle) = 0.$$

It means that the state $|1,0\rangle|1,1\rangle$ is transformed on a spinor representation of group SU (2). This result is an appearance of a cyclic symmetry of fermion multiple systems, meaning, that by virtue of identity of electrons there are the configurations which are produced by a cyclic permutation from a configuration with one unpaired electron (see fig. 1), including a configuration with "hole" - positron at electron with spin "down".

$$\{\underbrace{\uparrow\uparrow\uparrow\uparrow}_{k}|\underbrace{\downarrow\downarrow\downarrow\downarrow\downarrow}_{n-k}\}=\{\underbrace{\uparrow\uparrow\uparrow\downarrow}_{k}|\underbrace{\uparrow\downarrow\downarrow\downarrow\downarrow}_{n-k}\}+\{\underbrace{\uparrow\uparrow\uparrow\downarrow}_{k}|\underbrace{\downarrow\uparrow\downarrow\downarrow\downarrow}_{n-k}\}+\{\underbrace{\uparrow\uparrow\uparrow\downarrow}_{k}|\underbrace{\downarrow\downarrow\uparrow\downarrow\downarrow}_{n-k}\}+\ldots+\{\underbrace{\uparrow\uparrow\uparrow\downarrow}_{k}|\underbrace{\downarrow\downarrow\downarrow\downarrow\uparrow}_{n-k}\}$$

Fig. 1. Graphic representation of cyclic symmetry of wave function for electron

So, the secondary quantized wave function (1.4) is a wave function of electron without pair. It means, that Dirac hamiltonian (1.7) describes a secondary quantized wave function without pair. Hence, the procedure of secondary quantization (1.4) does not allow to describe process of pair production.

Further we shall develop a technique of projection operators allowing to secondary quantize a system with variable number of particles and pairs of particle - antiparticle.

3. Secondary quantized wave function of system with variable number of electron and fermionic pairs

Let us consider the quantum system consisting of variable (very large) number N of identical interacting particles $N \to \infty$. Its description will be complete if one knows accurate within phase multiplier $\exp(\imath\theta)$ a vector of state $|\phi_1>$ for one particle, a two-dimensional vector of state $|\phi_1,\phi_2>$ for a subsystem from two particles, a three-dimensional vector of state $|\phi_1,\phi_2,\phi_3>$ for a subsystem from three particles, etc. A wave function $|\hat{\phi}>$ of all multi-particle system is described by vectors with coordinates $<\hat{\phi}|\phi_0,\phi_1,\ldots,\phi_n>$ [Landau, Peierls]:

$$\begin{pmatrix} <\hat{\phi}|\phi_0><\phi_0| \\ <\hat{\phi}|\phi_0,\phi_1><\phi_0,\phi_1| \\ \ldots \\ <\hat{\phi}|\phi_0,\phi_1,\ldots,\phi_{n-1}><\phi_0,\phi_1,\ldots,\phi_{n-1}| \\ <\hat{\phi}|\phi_0,\phi_1,\ldots,\phi_n><\phi_0,\phi_1,\ldots,\phi_n| \end{pmatrix}. \qquad (3.1)$$

Here $|\phi_0,\phi_1,\ldots,\phi_n>$ is called a vector of state in vector Fock space; $\phi_0,\phi_1,\ldots,\phi_n$ are parameters of particles, for example, coordinates, momentum, energy. The secondary quantized function $|\hat{\phi}>$ consists of the sum of its projections:

$$<\hat{\phi}| = \sum_{n=0}^{\infty}\int\ldots\int d\phi_0\ldots d\phi_n <\hat{\phi}|\phi_0,\phi_1,\ldots,\phi_n><\phi_0,\phi_1,\ldots,\phi_n|, \qquad (3.2)$$

for a vacuum state ϕ_0 the following identity takes place

$$\int d\phi_0 \equiv 1. \tag{3.3}$$

Taking into account identity of particles we can define the one-particle annihilation operator $\widehat{\phi'_1}(\phi_\alpha)$ as

$$\widehat{\phi'_1}(\phi_\alpha) \equiv \widehat{\phi'_{k=1}}^{\dagger}\Big|_{\phi_n \to \phi_\alpha} = |\phi_0 >< \phi_\alpha| + \sqrt{2}\int d\phi_1 |\phi_0, \phi_1 >< \phi_0, \phi_1, \phi_\alpha| + \sqrt{3}\int d\phi_1 d\phi_2 |\phi_0, \phi_1, \phi_2 >< \phi_0, \phi_1, \phi_2, \phi_\alpha| + \ldots \tag{3.4}$$

and two-particle operators $\widehat{\Phi}^{\dagger}_{pair}$ and $\widehat{\Phi}_{pair}$ describing pairs of particles and given by following expressions

$$\widehat{\Phi}^{\dagger}_{pair} = \widehat{\phi'}_1(\phi_1)\widehat{\phi''}_1^{\dagger}(\overline{\phi}_1), \tag{3.5}$$

$$\widehat{\Phi}_{pair} = \widehat{\phi''}_1(\overline{\phi}_2)\widehat{\phi'}_1^{\dagger}(\phi_2), \tag{3.6}$$

Now it is possible to express the secondary quantized wave function through the introduced one-particle and two-particle operators

$$\hat{\phi}^{\dagger}_k = \widehat{\phi^{\dagger}}'^{-}_{1} \exp\left[\frac{(-1)}{2}\left(\Phi^{pair}\widehat{\Phi}^{+}_{pair} - \overline{\Phi}^{pair}\widehat{\Phi^{\dagger}}^{-}_{pair}\right)\right] = \widehat{\phi^{\dagger}}'^{-}_{1} \exp\left(\frac{\xi}{2}\widehat{\Phi}^{+}_{pair} - \frac{\eta}{2}\widehat{\Phi^{\dagger}}^{-}_{pair}\right). \tag{3.7}$$

The secondary quantized wave function (3.7) contains entangled states and consequently the positronic contribution at the offered way of quantization.

4. Method of Green functions

Let us utilize the technique of projection operators for the description of the Green function:

$$\hat{G}(z) = \hat{I}^2 \hat{G}(z) = \int\int d\vec{r} d\vec{r} \,|\, \vec{r} >< \vec{r} \,|\, \hat{G}(z) \,|\, \vec{r} >< \vec{r} \,| = \int\int d\vec{r} d\vec{r}' \,|\, \vec{r}' > G(\vec{r}, \vec{r}'; z) < \vec{r} \,|. \qquad (4.1)$$

In the secondary quantized case the operators $\hat{G}_N$, $\hat{H}_N$ become products of projection operators of a form

$$\hat{G}_N(t'_1 - t_1, \ldots, t'_N - t_N) = \int d\vec{r}_1 \ldots \vec{r}_N d\vec{r}\ _1 \ldots d\vec{r}\ _N \phi^+(\vec{r}\ _1, t_1) \ldots \phi^+(\vec{r}\ _N, t_N) \,|\, 0 >$$

$$\times G(\vec{r}_1, t_1; \ldots, \vec{r}_N, t_N; \vec{r}\ _1, , t_1; \ldots, \vec{r}\ _N, t_N) < 0 \,|\, \phi^-(\vec{r}_1, t_1) \ldots \phi^-(\vec{r}_N, t_N), \quad (4.2)$$

$$\hat{H}_N = \frac{1}{2} \sum_{i,j} \int d\vec{r}_i \vec{r}_j d\vec{r}\ _i d\vec{r}\ _j \times \phi^+(\vec{r}\ _i) \phi^+(\vec{r}\ _j) \,|\, 0 >< \vec{r}\ _i, \vec{r}\ _j \,|\, \hat{H}_1 \,|\, \vec{r}_i, \vec{r}_j > \delta(\vec{r}_i - \vec{r}\ _i) \delta(\vec{r}_j - \vec{r}\ _j) < 0 \,|\, \phi^-(\vec{r}_i) \phi^-(\vec{r}_j)$$

$$= \frac{1}{2} \sum_{i,j} \int d\vec{r}_i \vec{r}_j dt_j \times \phi^+(\vec{r}_i, t_i) \phi^+(\vec{r}_j, t_j) \,|\, 0 > \hat{H}_1(|\vec{r}_i - \vec{r}_j|) \delta(t_i - t_j) < 0 \,|\, \phi^-(\vec{r}_i, t_i) \phi^-(\vec{r}_j, t_j), \quad (4.3)$$

where the time $t_i(t'_i)$, $i = 1, \ldots, N$ is defined as $t_i = t + \varepsilon_i (t'_i = t' + \varepsilon'_i)$, $\varepsilon_i(\varepsilon'_i) \to 0$ and in this sense the equality $\phi^+(\vec{r}_i, t_i) = \phi^+(\vec{r}_i(t))$ $(\phi^-(\vec{r}_i, t_i) = \phi^-(\vec{r}_i(t)))$ is understood.

Knowing unperturbed Green function, the perturbed two-particle operator Green function can be found from the equation:

$$
\begin{aligned}
&(\widehat{\tilde{G}}_2)^{ns'ms} \overset{def}{=\!=} \phi^+(n)\phi^+(s')\phi^-(m)\phi^-(s) \\
&= (\phi^{(0)})^+(n)(\phi^{(0)})^+(s')(\phi^{(0)})^-(m)(\phi^{(0)})^-(s) + \frac{1}{2}\int dt_i dt_j d\vec{r}_i \vec{r}_j d\vec{r}\,'_i d\vec{r}\,'_j \times \\
&\times \delta(\vec{r}_i - \vec{r}\,'_i)\delta(\vec{r}_j - \vec{r}\,'_j)\Big[(\phi^{(0)})^+(j')(\phi^{(0)})^+(s')(\phi^{(0)})^-(m)(\phi^{(0)})^-(i') \\
&\times \phi^+(\vec{r}\,'_j, t'_j)\phi^+(\vec{r}\,'_i, t'_i)\hat{H}_1(|\vec{r}_i - \vec{r}_j|)\delta(t_i - t_j)\phi^-(\vec{r}_i, t_i)\phi^-(\vec{r}_j, t_j) \\
&\times \phi^+(n)\phi^+(i)\phi^-(j)\phi^-(s) + (\phi^{(0)})^+(j')(\phi^{(0)})^+(s')(\phi^{(0)})^-(m)(\phi^{(0)})^-(i') \\
&\times \phi^+(\vec{r}\,'_i, t'_i)\phi^+(\vec{r}\,'_j, t'_j)\hat{H}_1(|\vec{r}_i - \vec{r}_j|)\delta(t_i - t_j)\phi^-(\vec{r}_j, t_j)\phi^-(\vec{r}_i, t_i)\,\phi^+(n)\phi^+(i)\phi^-(j)\phi^-(s)\Big]. \qquad (4.4)
\end{aligned}
$$

Let us express $\phi^{+(-)}(i)$ through the one-particle operators $\phi_1^{+(-)}(i)$ and the operators $\exp\left(\frac{\xi}{2}\widehat{\Phi}^+_{pair} - \frac{\eta}{2}\widehat{\Phi^\dagger}^-_{pair}\right)$ describing processes of pair production. According to the expressions (4.4) the Green function is represented in the form

$$
\left\langle 0\right|(\widehat{\tilde{G}}_2)^{ns'n's}\left|0\right\rangle = \left\langle 0\right|\phi_1^+(n)e^{\zeta_n\widehat{\Phi}^+}\phi_1^+(s')e^{\zeta_{s'}\widehat{\Phi}^+}\phi_1^-(n')e^{-\bar{\zeta}_{n'}\widehat{\Phi}^-}\phi_1^-(s)e^{-\bar{\zeta}_s\widehat{\Phi}^-}\left|0\right\rangle. \qquad (4.5)
$$

Assuming $|0\rangle = |0\uparrow\rangle|0\downarrow\rangle$ and using a formula for multiplication of operator exponents

$$e^{\zeta_{s'}\hat{\Phi}^{+}} e^{\bar{\zeta}_{n'}\hat{\Phi}^{-}} = e^{\frac{1}{2}\zeta_{s'}\bar{\zeta}_{n'}[\hat{\Phi}^{+},\hat{\Phi}^{-}]} e^{\zeta_{s'}\hat{\Phi}^{+}-\bar{\zeta}_{n'}\hat{\Phi}^{-}} = e^{\zeta_{s'}\bar{\zeta}_{n'}\hat{\Phi}_{0}} e^{\zeta_{s'}\hat{\Phi}^{+}-\bar{\zeta}_{n'}\hat{\Phi}^{-}},$$

the expression (4.5) is rewritten as

$$\langle 0\uparrow|\langle 0\downarrow|(\hat{\tilde{G}}_{2})^{ns'n's}|0\downarrow\rangle|0\uparrow\rangle = \langle 0\uparrow|\phi_{1}^{+}(n)\phi_{1}^{+}(s')\phi_{1}^{-}(n')\phi_{1}^{-}(s)|0\uparrow\rangle\langle 0\downarrow|e^{(\zeta_{s'}\bar{\zeta}_{n'}+\zeta_{n}\bar{\zeta}_{s})\hat{\Phi}_{0}} e^{\zeta_{n}\hat{\Phi}^{+}-\bar{\zeta}_{s}\hat{\Phi}^{-}} e^{\zeta_{s'}\hat{\Phi}^{+}-\bar{\zeta}_{n'}\hat{\Phi}^{-}}|0\downarrow\rangle. \qquad (4.6)$$

Since $\Phi_0^2 = I$, then supposing $\bar{\zeta}_{s'} = -\bar{\zeta}_{s} = \bar{\zeta}_{n'}$, $\zeta_{n} = -\zeta_{n'}$, one has the following expansion

$$\exp(2|\zeta|^2\,\Phi_0) = \cos(2|\zeta|^2) + \Phi_0 \sin(2|\zeta|^2),$$

where $2|\zeta|^2 = 2\zeta_{s'}\bar{\zeta}_{n'}$.

Therefore the Green function (4.6) is transformed as

$$(\hat{\tilde{G}}_{2})^{ns'n's} = \phi_{1}^{+}(n)\phi_{1}^{+}(s')\phi_{1}^{-}(n')\phi_{1}^{-}(s)[\cos(2|\zeta|^2) + \Phi_0 \sin(2|\zeta|^2)]e^{(\zeta_{s'}\bar{\zeta}_{n'}+\zeta_{n}\bar{\zeta}_{s})\hat{\Phi}_{0}} e^{\zeta_{n}\hat{\Phi}^{+}-\bar{\zeta}_{s}\hat{\Phi}^{-}} e^{\zeta_{s'}\hat{\Phi}^{+}-\bar{\zeta}_{n'}\hat{\Phi}^{-}}. \qquad (4.7)$$

According to commutator properties of the operators Φ_{+}, Φ_{-} a state

$$N\exp\left(\zeta\Phi_{+} - \bar{\zeta}\Phi_{-}\right)|0\downarrow\rangle \equiv |\zeta\rangle = N\exp\left(\zeta\Phi_{+} - \bar{\zeta}\Phi_{-}\right)\left|\frac{1}{2}, -\frac{1}{2}\right\rangle$$

can be expanded over a orthonormal basis of states $\left|\frac{1}{2}, \sigma_z\right\rangle$, $\sigma_z = \pm 1/2$ [Perelomov]:

$$|\zeta\rangle = N\sum_{\sigma_z=-1/2}^{1/2} \zeta^{\frac{1}{2}+\sigma_z}\left|\frac{1}{2}, \sigma_z\right\rangle, \qquad N = \frac{1}{\sqrt{1+|\zeta|^2}}.$$

A relativistic invariant Green function is described component-wisely by such equation as Eq. (4.4) with additional trace operator Tr to sum over intermediate states with different spins:

$$(\widehat{\tilde{G}}_2)^{ns'ms}_{\alpha\beta\kappa\rho} = \phi^+_\alpha(n)\phi^+_\beta(s')\phi^-_\kappa(m)\phi^-_\rho(s)$$

which satisfies an Dyson – Schwinger equation.

In relativistic case a wave function is a four component bispinor:

$$\langle 0\downarrow|\exp\left(\zeta_n\hat{\Phi}^+ - \bar{\zeta}_s\hat{\Phi}\right)\exp\left(\zeta_{s'}\hat{\Phi}^+ - \bar{\zeta}_{n'}\hat{\Phi}\right)\begin{pmatrix}|\phi_\uparrow\rangle \\ |\phi_\downarrow\rangle\end{pmatrix} = \begin{pmatrix}\langle 0\downarrow|0\uparrow\rangle \\ |\zeta|^2\end{pmatrix} = \langle 0\downarrow|\begin{pmatrix}\left|\frac{1}{2},\frac{1}{2}\right\rangle \\ |\zeta|^2\left|\frac{1}{2},-\frac{1}{2}\right\rangle\end{pmatrix} \qquad (4.8)$$

and the operator Φ_0 passes into an operator τ_0:

$$\tau_0 = \gamma_5\alpha_3 = \begin{pmatrix}0 & I \\ I & 0\end{pmatrix}\begin{pmatrix}0 & \Phi_0 \\ \Phi_0 & 0\end{pmatrix} \equiv \begin{pmatrix}\sigma_3 & 0 \\ 0 & \Phi_0\end{pmatrix},$$

where τ_0 is a Z component of relativistic spin operator.

When $\zeta \to 0$ one has large upper components of the bispinor (4.8). In this nonrelativistic limit $\zeta \to 0$ polarized contribution enters into the Green function as a correction:

$$(\widehat{\tilde{G}}_2)^{ns'n's} = \phi_1^+(n)\phi_1^+(s')\phi_1^-(n')\phi_1^-(s) + 2|\zeta|^2\,\phi_1^+(n)\phi_1^+(s')\phi_1^-(n')\phi_1^-(s)\tau_0\,.$$

$$\zeta \to 0 \quad (4.9)$$

and we have a known problem with nonpaired Dirac hamiltonian.

When $2|\zeta|^2=\frac{\pi}{2}+2\pi k\gg 1$ (k is integer) one has large bottom components of the bispinor. The problem is a multi -particle one. In this second limit case a self-consistent Hartry – Fock – Dirac approximation is inapplicable, as a correlation interaction (polarization contribution) is large enough in comparison with exchange. Let us find a solution of the problem for multi -particle dynamics.

5. Strongly correlated systems

Since $\sin(2|\zeta|^2)\approx 1$, a Green function of strongly correlated system has the following form

$$(\widehat{G}_2)^{ns'n's} = \tau_0(\widehat{\widetilde{G}}_2)^{ns'n's} = (|\zeta|^{1/2}\,\tau_0\phi_1^+(n)\tau_0)(|\zeta|^{1/2}\,\tau_0\phi_1^+(s')\tau_0)(|\zeta|^{1/2}\,\tau_0\phi_1^-(n')\tau_0)(|\zeta|^{1/2}\,\tau_0\phi_1^-(s)\tau_0)\,.$$

In representation of Wigner functions a retard Green function satisfies the following equation

$$\imath\hbar\left(\frac{1}{2}\partial_T+\frac{\imath}{\hbar}E+\frac{\imath\vec{p}^{\,2}}{2m\hbar}+\frac{\vec{p}}{2m}\cdot\vec{\partial}_R-\frac{\imath\hbar}{8m}\vec{\partial}_R^{\,2}\right)g^<(p,R)-\Sigma^+(p,R)g^<(p,R)\;-\Sigma^<(p,R)g^+(p,R)=\Sigma^>(p,R)g^<(p,R)-\Sigma^<(p,R)g^>(p,R).\quad(5.1)$$

Here one introduces the following notation

$$\Sigma^i(p,R)g^j(p,R)=\int d^4r e^{-\frac{\imath pr}{\hbar}}\int dx\Sigma^i(R+\frac{1}{2}r,x)g^j(x,R-\frac{1}{2}r),\qquad i,j=+,<,>,\;\; p=\{E,\vec{p}\},\;\; R=\{T,\vec{R}\}\;\;(5.2)$$

Condition that $p=\{\vec{p},E\}>1$ and taking into account that a resonant contribution from causal Green functions is an essential one:

$$g^+(p,R)=\left[E-\frac{\vec{p}^{\,2}}{2m}-\Sigma^+(p,R)+\imath\varepsilon\right]^{-1},$$

lead to separation of variables in the kinetic equation (5.1). In a gradient approximation, it is transformed to the form

$$\imath\hbar\frac{1}{2}\partial_T g^<(p,R)-\frac{1}{2}\imath\hbar\left\{(\Re e\Sigma^+(p,R)),g^<(p,R)\right\}-\frac{1}{2}\imath\hbar\left\{\Sigma^<(p,R),(\Re e\, g^+(p,R))\right\}=\Sigma^>(p,R)g^<(p,R)-\Sigma^<(p,R)g^>(p,R);\quad p\neq 0.$$

$$(5.3)$$

$$\left(\frac{1}{\hbar}\Re e\Sigma^+(p,R))+\frac{\vec{p}^{\,2}}{m\hbar}-\frac{\hbar}{8m}\vec{\partial}_R^{\,2}\right)g^<(p,R)=0,\qquad\qquad g^<(p,R)=|\zeta|(\vec{p},T)g^<(\vec{R}).$$

Here {,} is a Poisson bracket.

Let us examine approximation by which the real part $\Re e\,\Sigma^{+}(p,R)$ of self-energy for the multi -particle system with a particle pair is equal to a real part $\Re e\,\Sigma_1^{+}(\vec{R})$ of self-energy for one-particle state minus a kinetic energy of the pair:

$$\Re e\,\Sigma^{+}(p,R) = \Re e\,\Sigma_1^{+}(\vec{R}) - \frac{\vec{p}^{\,2}}{m}. \tag{5.4}$$

In this approximation the causal Green function

$$g^{+} = \left[E - \frac{\vec{p}^{\;2}}{2(-m)} - \Sigma_1^{+}(\vec{R}) + \imath\varepsilon \right]^{-1} = g^{+}{}_{hole}$$

is a causal function of antiquasi-particle – hole. Since

$$g^{+}{}_{hole} \approx const\,,$$

the system of equations (5.3) is transformed to the form

$$\imath\hbar\frac{1}{2}\partial_T g^{<}(p,R) - \frac{1}{2}i\hbar\left\{\Re e\left(\Sigma_1^{+}(\vec{R}) - \vec{p}^{\,2}/m\right), g^{<}(p,R)\right\} = \Sigma^{>}(p,R)g^{<}(p,R) - \Sigma^{<}(p,R)g^{>}(p,R); \quad p \neq 0.$$

$$\left(\frac{1}{\hbar}\Re e\,\Sigma_1^{+}(\vec{R}) - \frac{\hbar}{8m}\vec{\partial}_R^{\,2}\right) g^{<}(p,R) = 0\,, \qquad\qquad g^{<}(p,R) = |\,\zeta(\vec{p},T))\,|\, g^{<}(\vec{R}). \tag{5.5}$$

In case under consideration when the contribution of multi-particle effects is large, the self-energy Σ_1^+ is approximately equal to the correlation energy:

$$\Sigma_1^+ \approx \Sigma_1^{+(c)},$$

because the exchange contribution $\Sigma_1^{+(x)}$ is small. Since the correlation energy is of pronounced oscillatory character (Friedel oscillations), the derivative

$$\frac{\partial}{\partial \vec{R}} g^{<}(\vec{R}) = \int \frac{8m}{\hbar^2} \Re e\, \Sigma_1^{+(c)}(\vec{R})\, d\vec{R} \approx 0\,.$$

is approximately equal to zero.

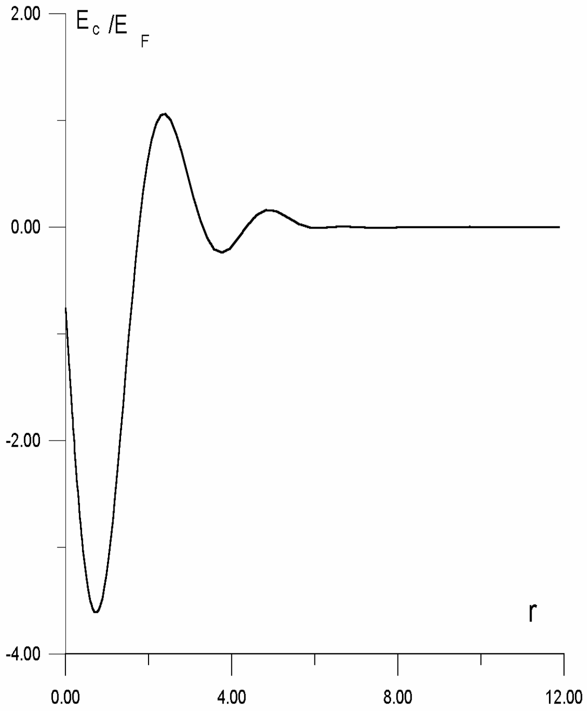

FIG. 2. A dependence of real part of correlation interaction potential $E_c(r)$ on distance r. E_F is a Fermi energy.

Hence, we get finally the following kinetic equation for $|\zeta|$

$$\imath\hbar\frac{1}{2}\left(\partial_T+\frac{\vec{p}}{m}\cdot\vec{\nabla}_R\right)|\zeta(\vec{p},T))|-\frac{1}{2}i\hbar\vec{\nabla}_R\Re e\Sigma_1^+(\vec{R})\cdot\vec{\nabla}_p|\zeta(\vec{p},T))|=(\Sigma^>(p,R)-\Sigma^<(p,R)g^>(\vec{R})/g^<(\vec{R}))|\zeta(\vec{p},T))|;\quad p\neq 0.$$

and a wave equation for $g^<(\vec{R})$. The equation for the parameter ζ looks like to a classical Boltzmann equation. This is a consequence of the fact that coherent states are most proximate to classical ones.

CONCLUSIONS

- A secondary quantized wave function of fermion multiple system is found in terms of one-particle fermionic operators of creation (annihilation) and two-particle operators of creation (annihilation).
- The method of Green functions is proposed for quantum-field description of a problem of pair production.
- The contribution of the processes of quasi-particle pair production into coherent charge transfer in strongly correlated systems is estimated.