Top Quark Decay Analysis including QED and QCD radiative corrections

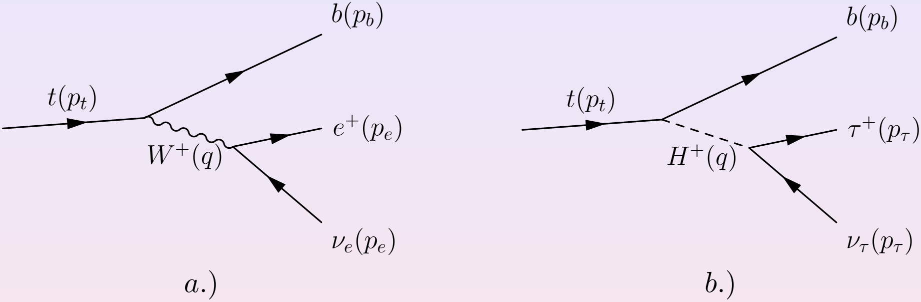
A. Ali[1], E.A. Kuraev[2], Yu.M. Bystritskiy[2]

[1] DESY, Hamburg, Germany
[2] Bogoliubov Laboratory of Theoretical Physics, JINR Dubna, Russia

ISMD-2009
5 September 2009

Talk plan

1. Top quark decay mode $t \to bW^+ \to b(\ell^+ \nu_\ell)$
    1. Born approximation
    2. QCD radiative corrections
    3. QED radiative corrections
    4. QED radiative corrections: Kinoshita-Lee-Nauenberg theorem
2. Top quark decay mode $t \to bH^+ \to b(\tau^+ \nu_\tau)$
    1. Born approximation
    2. QED radiative corrections
    3. QCD Radiative Corrections: Details of calculcation
3. Radiative corrections to both top quark decay modes $t \to b(W^+, H^+) \to b(\tau^+ \nu_\tau)$
4. Radiative corrections contribution to lepton universality
5. Conclusions



Matrix element of $t \to bW^+ \to b(\ell^+\nu_\ell)$ within the Standard Model (SM):

$$M_{\rm Born}^{t\to bW^+\to b(e^+\nu_e)} = i\frac{g^2 V_{tb}}{4\sqrt{2}} \frac{1}{q^2 - M_W^2 + iM_W\Gamma_W} \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_W^2} \right) \times$$
$$\times \quad [\bar{u}_b\,(p_b)\,\gamma^\mu\,(1+\gamma_5)\,u_t\,(p_t)]\,[\bar{u}_e\,(p_e)\,\gamma^\nu\,(1-\gamma_5)\,u_{\nu_e}\,(p_\nu)]\,, \qquad (1)$$

Top Quark Decays: Born approximation

The kinematical notations:

$$x_b = \frac{2E_b}{M_t}, \qquad x_e = \frac{2E_e}{M_t}, \qquad x_\nu = \frac{2E_\nu}{M_t}, \qquad y = 1 + \eta - x_b,$$
$$\gamma = \frac{\Gamma_W}{M_W}, \qquad \xi = \frac{M_t^2}{M_W^2}, \qquad \eta = \frac{M_b^2}{M_t^2}, \qquad x_e^{max} = 1 - \eta,$$

The decay spectrums:

$$\frac{d\Gamma_{\rm Born}^{t\to bW^+\to b(l^+\nu_l)}}{dx_b dx_l} = \Gamma_t \frac{x_l\left(x_l^{max} - x_l\right)}{\left(1 - \frac{y}{y_0}\right)^2 + \gamma^2}, \tag{2}$$

$$\frac{d\Gamma_{\rm Born}^{t\to bW^+\to b(e^+\nu_e)}}{dx_e} = \int\limits_{1-x_e}^{1} dx_b \frac{d\Gamma_{\rm Born}^{t\to bW^+\to b(e^+\bar\nu_e)}}{dx_b dx_e} = \Gamma_t \cdot x_e\left(x_e^{max} - x_e\right)\Phi_W\left(x_e\right), \tag{3}$$

where

$$\Phi_W(x) = \int\limits_0^x \frac{dy}{\left(1 - \frac{y}{y_0}\right)^2 + \gamma^2} =$$
$$= \frac{1}{\gamma\xi}\left[\arctan\left(\frac{\xi\left(1 - \sqrt{\eta}\right)^2 - 1}{\gamma}\right) + \arctan\left(\frac{\xi\left(\eta + x\right) - 1}{\gamma}\right)\right]. \tag{4}$$

Top Quark Decays: QCD Radiative Corrections

The inclusive electron energy spectrum including the lowest order QCD corrections is

$$\frac{d\Gamma^{t\to bW^+\to b(e^+\nu_e)}_{Born+QCD}}{dx_e} = \Gamma_t \int\limits_0^{x_e} \frac{dy}{(1-\xi y)^2+\gamma^2}\left[x_e\left(x_e^{max}-x_e\right) - \frac{2\alpha_s}{3\pi} F_W\left(x_e, y\right)\right]. \qquad (5)$$

where the function $F_W(x, y)$ is finite in the limit $M_b \to 0$ and has the form (M. Jezabek and J. H. Kühn, Nucl. Phys. **B320**, 20 (1989)):

$$\begin{aligned} F_W(x,y) &= 2x(1-x)\left[\zeta_2 + \mathrm{Li}_2(x) + \mathrm{Li}_2\left(\frac{y}{x}\right) + \frac{1}{2}\ln^2\left(\frac{1-y/x}{1-x}\right)\right] + \\ &+ x\left[\zeta_2 + \mathrm{Li}_2(y) - \mathrm{Li}_2(x) - \mathrm{Li}_2\left(\frac{y}{x}\right)\right] + \\ &+ \frac{1}{2}\ln(1-y)\left[-(3+2x)+2y(1+x)+y^2\right] + \\ &+ \frac{1}{2}\ln\left(1-\frac{y}{x}\right)\left[x(9-4x)-2y(1+x)-y^2\right] + \\ &+ \frac{5(1-x)}{2}\ln(1-x) + \frac{1}{2}y(1-x)\left(\frac{y}{x}+4\right). \qquad (6) \end{aligned}$$

This formula is valid for $x_e < 1$. The case $x_e \approx 1$ where this result becomes unstable is practically not interesting.

Top Quark Decays: QED Radiative Corrections

To calculate the QED radiative corrections in the leading logarithmic approximation we will use the **Structure Function (SF) Method**. The QED radiative corrected spectrum is:

$$\frac{d\Gamma^{t\to bW^+\to b(e^+\bar{\nu}_e)}_{Born+QED}}{dx_e} = \int_{x_e}^{1} \frac{dy_e}{y_e} D\left(\frac{x}{y_e}, \beta_e\right) \frac{d\Gamma^{t\to bW^+\to b(e^+\bar{\nu}_e)}_{B}}{dy_e}, \qquad (7)$$

where the structure function $D(x, \beta_e)$ is (E. A. Kuraev and V. S. Fadin, Sov. J. Nucl. Phys. **41**, 466 (1985))

$$D(x, \beta_e) = \delta(1-x) + \beta_e P^{(1)}(x) + \frac{1}{2!}\beta_e^2 P^{(2)}(x) + \cdots, \qquad (8)$$

$$\beta_e = \frac{\alpha}{2\pi}(L_e - 1), \qquad L_e = \ln\left(\frac{M_t^2}{m_e^2}\right) \approx 25.4, \qquad (9)$$

$P^{(n)}(x)$ are the kernels of the evolution equations:

$$P^{(1)}(x) = \left(\frac{1+x^2}{1-x}\right)_+ = \lim_{\Delta\to 0}\left[\frac{1+x^2}{1-x}\theta(1-x-\Delta) + \left(2\ln(\Delta) + \frac{3}{2}\right)\delta(1-x)\right], \qquad (10)$$

$$P^{(n)}(x) = \int_{x}^{1} \frac{dy}{y} P^{(1)}(y) P^{(n-1)}\left(\frac{x}{y}\right).$$

The structure function $D\left(x, \beta_e\right)$ defined in this way automatically satisfies the Kinoshita-Lee-Nauenberg (KLN) theorem (T. Kinoshita, J. Math. Phys. **3**, 650 (1962); T. D. Lee and M. Nauenberg, Phys. Rev. **133**, B1549 (1964)) of the cancellation of the mass singularities in the total decay width

$$\int\limits_0^1 dy \int\limits_y^1 \frac{dt}{t} D\left(\frac{y}{t}, \beta_e\right) F(t) = \int\limits_0^1 F(t) dt.$$

$$\beta_e = \frac{\alpha}{2\pi}\left(L_e - 1\right), \qquad L_e = \ln\left(\frac{M_t^2}{m_e^2}\right).$$

$$\frac{d\Gamma^{t\to bW^+\to b(e^+\bar{\nu}_e)}_{QED_{LLA}}}{dx_e} = \frac{\alpha}{2\pi}\left(L_e-1\right)\int\limits_{x_e}^{1}\frac{dy_e}{y_e}P^{(1)}\left(\frac{x_e}{y_e}\right)\frac{d\Gamma^{t\to bW^+\to b(e^+\bar{\nu}_e)}_{B}}{dy_e}$$
$$= \frac{\alpha}{2\pi}\left(L_e-1\right)\Gamma_t\int\limits_{x_e}^{1}\frac{dy_e}{y_e}P^{(1)}\left(\frac{x_e}{y_e}\right)y_e\left(y_e^{max}-y_e\right)\Phi\left(y_e\right)$$
$$= \frac{\alpha}{2\pi}\left(L_e-1\right)\Gamma_t\ I\left(x_e\right), \tag{11}$$

where $y_e^{max} = 1-\eta$, and

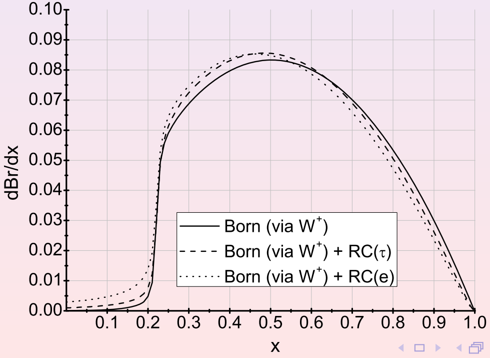
$$I\left(x\right) = \int\limits_{x}^{1}\frac{dy}{y}P^{(1)}\left(\frac{x}{y}\right)y\left(y^{max}-y\right)\Phi\left(y\right) =$$
$$= \Phi\left(x\right)\left\{x\left(1-x\right)\left[2\ln\left(\frac{1-x}{x}\right)+\frac{3}{2}\right]+x\ln\left(x\right)+\left(1-x\right)^2-\frac{1}{2}\left(1-x^2\right)\right\}+$$
$$+ \int\limits_{x}^{1}dy\frac{\left(1-y\right)\left(y^2+x^2\right)}{y\left(y-x\right)}\left[\Phi\left(y\right)-\Phi\left(x\right)\right]. \tag{12}$$

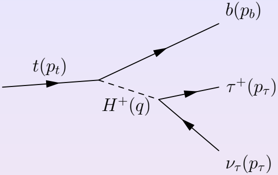
For the experimental setup in which both the gluonic jets and the electron are measured:

$$\frac{d\Gamma^{t\to bW^+\to b(l^+\nu_l)}}{dx_b dx_l} = \int_{x_l}^{1} \frac{dy_l}{y_l} \int_{x_b}^{1} \frac{dy_b}{y_b} \frac{d\Gamma_B^{t\to bW^+\to b(l^+\bar{\nu}_l)}}{dy_b dy_l} \times$$

$$\times \quad D^{QCD}\left(\frac{x_b}{y_b}, \beta_b\right) D_l^{QED}\left(\frac{x_l}{y_e}, \beta_l\right)\left(1 - \frac{2\alpha_s}{3\pi} F_W\left(y_l, y_b\right)\right), \quad (13)$$

where $\beta_b = \frac{\alpha}{2\pi}(L_b - 1)$ and $\beta_l = \frac{\alpha}{2\pi}(L_l - 1)$.





Lagrangian in the MSSM (and in the Two-Higgs-Doublet Model (2HDM)) is:

$$\mathcal{L}_{Int} = \frac{g}{2\sqrt{2}M_W} V_{tb} H^+ \left[\bar{u}_t\left(p_t\right)\left\{A\left(1+\gamma_5\right)+B\left(1-\gamma_5\right)\right\}u_b\left(p_b\right)\right] + + \frac{gC}{2\sqrt{2}M_W} H^+ \left[\bar{u}_{\nu_l}\left(p_\nu\right)\left(1-\gamma_5\right)u_l\left(p_l\right)\right], \quad (14)$$

where H^+ stands for charged Higgs field, and A, B and C are the model-dependent parameters, which depend on the fermion masses and $\tan\beta$:

$$A = M_t \cot\beta, \qquad B = M_b \tan\beta, \qquad C = M_\tau \tan\beta, \quad (15)$$

The matrix element of the process $t \to bH^+ \to b(l^+\bar{\nu}_l)$ is

$$M_B^{t\to bH^+\to b(l^+\nu_l)} = i\frac{g^2 V_{tb}}{8M_W^2}\frac{C}{q^2 - M_H^2 + iM_H\Gamma_H}\left[\bar{u}_{\nu_l}\left(p_\nu\right)\left(1+\gamma_5\right)u_l\left(p_l\right)\right] \times \times \left[\bar{u}_t\left(p_t\right)\left\{A\left(1-\gamma_5\right)+B\left(1+\gamma_5\right)\right\}u_b\left(p_b\right)\right]. \quad (16)$$

QED Radiative Corrections: First Order RC in Leading Logarithmical Approximation

To calculate the QED radiative corrections in the leading logarithmic approximation we will again use the **Structure Function Method**:

$$\frac{d\Gamma^{t\to bH^+\to b(\ell^+\nu_\ell)}_{Born+QED}}{dx_\ell} = \int_{x_\ell}^{1} \frac{dy}{y} D\left(\frac{x_\ell}{y}, \beta_l\right) \frac{d\Gamma^{t\to bH^+\to b(l^+\nu_l)}_{Born}}{dy}. \quad (17)$$

where the large QED logarithm now is $\beta_\ell = \beta_e, \beta_\mu, \beta_\tau$

$$\beta_\ell = \frac{\alpha}{2\pi}(L_\ell - 1), \qquad L_e = \ln\left(\frac{M_t^2}{m_e^2}\right) \approx 25.4,$$

$$L_\mu = \ln\left(\frac{M_t^2}{m_\mu^2}\right) \approx 14.8, \qquad L_\tau = \ln\left(\frac{M_t^2}{m_\tau^2}\right) \approx 9.1.$$

The first order QED radiative corrections then

$$\frac{d\Gamma^{t\to bH^+\to b(l^+\nu_l)}_{QED_{LL}}}{dx_l} = \frac{\alpha}{2\pi}(L_l - 1)\int_{x_l}^{1} \frac{dy}{y} P^{(1)}\left(\frac{x_l}{y}\right) \frac{d\Gamma^{t\to bH^+\to b(l^+\nu_l)}_{Born}}{dy} =$$

$$= \frac{\alpha}{2\pi}(L_l - 1)\,\Gamma_t\; I_H(x_l). \quad (18)$$

Soft gluon emission, i.e. $\omega < \Delta E_b \ll M_t$ (we work in the rest frame of top quark) leads to

$$\frac{d\Gamma_{soft}}{d\Gamma_{Born}} = -\frac{\alpha}{4\pi^2}\int\frac{d^3k}{\omega}\left(\frac{p}{pk}-\frac{p_b}{p_bk}\right)^2 = \frac{\alpha}{\pi}\left[2(l-1)\ln\frac{2\Delta E_b}{\lambda}+1+l-l^2-\frac{\pi^2}{6}\right], \qquad l=\ln\frac{yM_t}{M_b}. \tag{19}$$

Soft and virtual corrections contributions:

$$d\Gamma_{S+V} = d\Gamma_{Born}\left\{1+\frac{\alpha}{2\pi}(L-1)\left(2\ln\Delta+\frac{3}{2}\right)+ \frac{\alpha}{\pi}\left[-\ln\Delta-\frac{5}{2}+\frac{1}{2}\ln^2 y-\frac{5}{2}\ln y+Li_2\left(1-\frac{1}{y}\right)-\frac{\pi^2}{6}-\frac{1}{1-y}\ln y\right]\right\}, \tag{20}$$

where $\Delta=\Delta E_b/E_b$, $L=\ln\frac{M_t^2y^2}{M_b^2\Delta}$. Note that the term containing $\ln\Delta$ are connected with emission from the "light" b-quark and the heavy t-quark.

Hard photon emission (here we present the contributions of only collinear region ($\vec{k}||\vec{p}_b$):

$$d\Gamma_{coll}=\frac{\alpha}{2\pi}\int\limits_{y(1+\Delta)}^{1}\frac{dt}{t}d\Gamma_{Born}(t)\left[\frac{1+\frac{y^2}{t^2}}{1-\frac{y}{t}}(L_\sigma-1)+1-\frac{y}{t}\right], \tag{21}$$

where σ is some auxiliary parameter which separates the region of collinerity and

$$L_\sigma=\ln\frac{M_t^2y\sigma}{xM_b^2}, \qquad \Delta=\frac{2\Delta E}{M_ty}.$$

QCD Radiative Corrections: Gather the final result

Gathering together all terms of QCD RC (and performing the substitution $\alpha \to \alpha_s C_F$, $C_F = \frac{N_c^2-1}{2N_c} = \frac{4}{3}$) we obtain the following expression for x_τ and y distribution:

$$\frac{d\Gamma_{Born+QCD}}{dx_\tau dy} = \int\limits_y^1 \frac{dt}{t} D^{QCD}\left(\frac{y}{t},\beta\right) \frac{d\Gamma_{Born}(t,x_\tau)}{dtdx_\tau}\left(1+\frac{\alpha_s C_F}{\pi}K_H\right), \quad (22)$$

where K_H is the K-factor which contains all the non-enhanced terms.

$$\frac{d\Gamma_{Born+QCD}}{dtdx_\tau} = \frac{d\Gamma_{Born}}{dtdx_\tau} - \frac{2\alpha_s}{3\pi}\int\limits_0^{x_\tau} dy \frac{d\Gamma_{Born}}{dydx_\tau} F_H(x_\tau, y) = \quad (23)$$

$$= \frac{d\Gamma_{Born}}{dtdx_\tau} - \frac{\alpha_s C_F}{\pi}\int\limits_{t(1+\Delta)}^1 dy \frac{d\Gamma_{Born}}{dydx_\tau}\frac{y^2}{t^2}\frac{1}{t-y} +$$

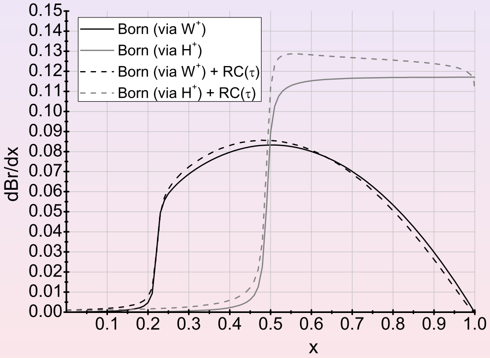
$$+ \frac{\alpha_s C_F}{\pi}\frac{d\Gamma_{Born}}{dtdx_\tau}\left[-\frac{5}{2}+\frac{1}{2}\ln^2 t - \frac{5}{2}\ln t - \frac{\ln t}{1-t} - \zeta_2 + \mathrm{Li}_2\left(1-\frac{1}{t}\right) - \frac{t}{2} - \ln\Delta\right].$$

This contributions do not depend on the value of small auxiliary parameter $\Delta \ll 1$.

The tau-lepton differential brenching for the process

$$t\,(p_t) \to b\,(p_b) + \left\{W^+\,(q)\,, H^+\,(q)\right\} \to b\,(p_b) + \tau^+\,(p_\tau) + \nu_\tau\,(p_\nu) \qquad (24)$$

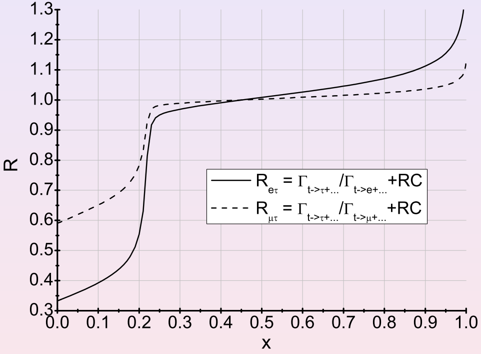
where only the energy fraction of final tau-lepton $(x = 2E_\tau/M_t)$ is measured have a form



We used rather large value of $\tan\beta \approx 40$ (the Two-Higgs-Doublet Model parameter) in order to provide the total brenching of decay mode with H^+ in the intermediate state of order $\sim 10\%$.

Top Quark Decays: Lepton universality check

Radiative corrections give noticeable contribution into the ratios of the lepton-energy spectra in the decays $t \to bW^+ \to b(\ell^+\nu_\ell)$ quantifying the breakdown of the lepton universality.



The solid curve shows the ratio $R_{e\tau}(x) = \frac{d\Gamma^{t\to bW^+\to b(\tau\nu_\tau)}}{dx_\tau} / \frac{d\Gamma^{t\to bW^+\to b(e\nu_e)}}{dx_e}$, and the dashed curve is the ratio $R_{\mu\tau}(x) = \frac{d\Gamma^{t\to bW^+\to b(\tau\nu_\tau)}}{dx_\tau} / \frac{d\Gamma^{t\to bW^+\to b(\mu\nu_\mu)}}{dx_\mu}$.

Conclusions

1. We calculated the QED radiative corrections to the following top quark decay modes $t \to bW^+ \to b(\ell^+ \nu_\ell)$ and $t \to bH^+ \to b(\tau^+ \nu_\tau)$ within the **Structure Function Approach**.
2. We calculated the QCD radiative corrections to $t \to bH^+ \to b(\tau^+ \nu_\tau)$ in the first order of perturbation theory and summed logarithmically enhanced terms in all orders of perturbation theory using the **Structure Function Approach**.
3. The contributions of radiative corrections into lepton universality was evaluated.